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VIII. *The Law of Plastic Flow of a Ductile Material and the Phenomena of Elastic and Plastic Strains.* *By* CHARLES EDWARD LARARD.

RECEIVED NOVEMBER 1, 1912. READ NOVEMBER 22, 1912.

IN a Paper recently read before Section G of the British Association,* the author, *en passant*, stated briefly certain conclusions he had arrived, at connecting the variables, torque, time and twist, in the case of a ductile material which had been twisted to destruction. In the present Paper he offers a sufficiently full account of the investigation on a piece of mild steel of the form and dimensions shown in Fig. 1, which was twisted to destruction under a pure torque of increasing magnitude.

It is a matter of common experience to those who have to deal experimentally with the straining of material to destruction that the load, and, in the case of torsion, the twisting

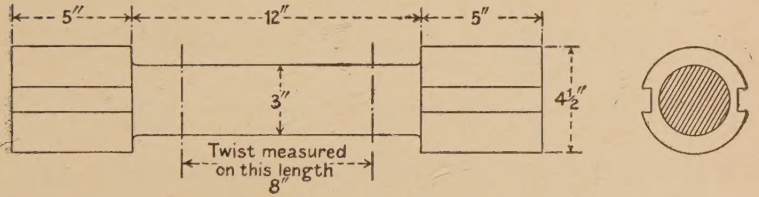


FIG. 1.

moment, must be increased more and more slowly, or at a diminishing rate of increase with respect to time, as the test proceeds, and that towards the end of the test, when the material is reaching its breakdown, very small increments in the load or torque are accompanied by very large amounts of plastic or permanent strain.

Three principal cases presented themselves to the author before commencing the experimental work. These are the cases where—

1. The angular velocity of straining or twisting is maintained constant, the torque being increased continuously so as to maintain equilibrium between the twisting moment and the moment of resistance of the specimen.

* Dundee Meeting, September, 1912.

2. The torque-time rate (or, as it may be termed, the velocity of the torque) is kept constant and the twist-time rate being continuously increased to maintain equilibrium.

3. After each step-by-step increment in the torque time is allowed for the greater part of the plastic strain to be recorded before the next increase in the magnitude of the torque.

The author has made many experiments in connection with all three, but he here presents a partial solution of the first

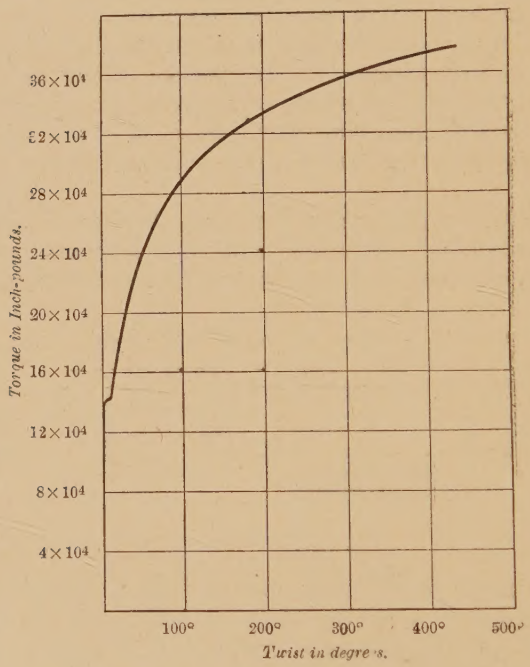


FIG. 2.—AUTOGRAPHIC TORQUE TWIST DIAGRAM (FOR 12 IN. LENGTH BETWEEN SHOULDERS.)

problem only, in the particular case where the specimen was twisted to destruction at a uniform angular velocity, ω, of 0·1145 deg. per second. For the purpose of this and other tests three special instruments or pieces of apparatus were used :—

1. A recorder to draw autographically throughout the test the torque-twist curve for the full 12 in. length of specimen between the shoulders. This curve is reproduced in Fig. 2;

and shows clearly the kind of curve obtained with a yield point very clearly defined.

2. A torsion indicator with a large graduated aluminium circle for determining the angle of twist, at any stage in the test, on an 8 in. length of the specimen, being the distance between the gauge points.

3. Apparatus for simultaneously marking the time on the torsion circle, the steelyard scale and the autographic diagram, so that both the torque-time and twist-time rates as well as the corresponding accelerations could be deduced.

The material for the purpose of these tests was supplied by Messrs. Firth, of Sheffield. The specimen was forged at an approximate temperature of 850°C. as a cylindrical piece $4\frac{3}{4}$ in. diameter from billets of Siemens-Martin mild steel $1\frac{3}{4}$ in. round. It was then annealed in an oven for one hour at a temperature of 750°C., after which is was removed, buried in sand, and allowed to cool gradually. After cooling it was machined to the dimensions given in Fig. 1.

A chemical analysis of drillings taken from the specimen gave the following composition :—

Carbon	0·320 per cent.
Silicon	0·160 ,, ,,
Manganese	0·530 ,, ,,
Sulphur	0·028 ,, ,,
Phosphorus	0·030 ,, ,,

The cost of the steel and its machining was defrayed out of a Government grant received through the Royal Society.

The twisting of the specimen was carried out in the large testing machine* installed in the author's laboratory at the Northampton Institute, London.

It will be sufficient for the purpose of this Paper to state briefly the manner of holding and twisting the specimen and applying the torque. One enlarged end of the torsion specimen is inserted into a die-block sliding into a bracket fixed to the steelyard lever of the testing machine, while the other enlarged end is held by a die which slides into the boss of a large circumferentially toothed straining wheel connected by gearing to an electric motor. The two end dies and attachments ensure that the geometrical axis of the specimen when produced shall

* For descriptions *see* the author's Papers in the "Proceedings" of the Institution of Mechanical Engineers, Vols. III. and IV., 1907, and the British Association Paper published in "Engineering," on Sept. 3 and 10, 1909.

be coincident with the line of contact of the knife edge of the steelyard lever with its support. The rotation of the large wheel effected the twisting of the specimen. To keep the steelyard lever horizontal so that there should be equilibrium between the torque and the resisting moment, the poise weight was moved forward in the positive direction, defining by its position on the steelyard scale the magnitude of the torque.

Before starting the test, the specimen was placed in the machine and the vernier scale attached to the poise weight adjusted to zero on the steelyard scale, with the beam floating horizontally in its neutral position. At a given moment, the time being noted, the electric motor was started and kept running throughout the test, driving the gear and twisting the

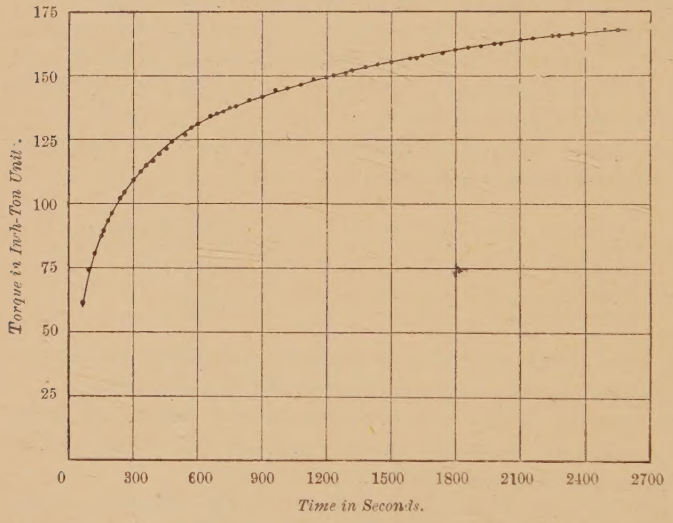


FIG. 3.—TORQUE TWIST CURVE FOR STEEL SPECIMEN, 3 IN. DIAMETER BY 8 IN. LONG.

specimen at the uniform rate stated. The poise weight was advanced continuously so as to keep the lever (after the yield period was passed) in its horizontal position throughout the test, and time intervals were marked simultaneously at predetermined times on the steelyard scale, the torsion circle and the autographic diagram. For the purpose of receiving the time marks, a specially prepared paper was used, glued by fish glue both to the aluminium torsion circle and to the steelyard scale on the lever. The pencils for marking time were

pieces of pointed brass wire which, on the depression of a key at required intervals, came into contact with the paper.

During the testing to destruction of this specimen some hundreds of observations of time, torque and twist were made. To publish all the tabular data would make a very bulky Paper. The author has deemed it advisable, therefore, after reducing the results, to give his deductions in graphical and mathematical form.

Some of the plottings for the torque, T time t, and the twist θ for the 8 in. length are given in Fig. 3, and the curve obtained gives a good idea of the kind of variation between these two variables. The experimental values of $\frac{\Delta t}{\Delta T}$ obtained from the tabular data have been plotted as ordinates to a time

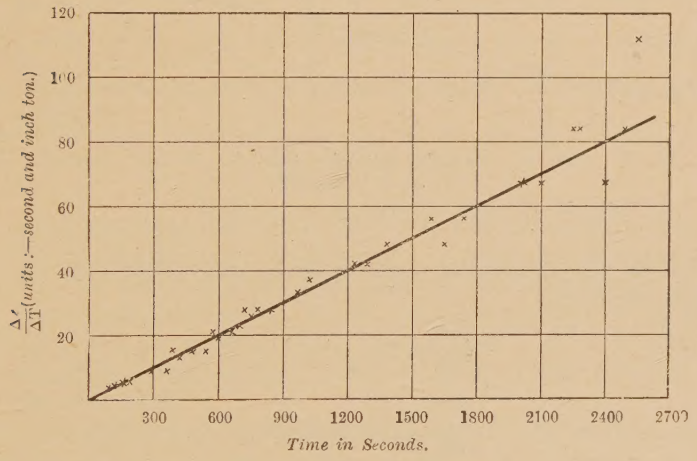


FIG. 4.

base (*see* Fig. 4) and an inspection of the result indicates that there is a linear relationship between these two variables with, in the limit, assuming the line to pass through the origin, an equation of the form

$$\frac{dt}{d\mathrm{T}} = bt \quad \text{or} \quad \frac{d\mathrm{T}}{dt} = \frac{1}{bt}.$$

Consequently $\frac{d\mathrm{T}}{dt} \times t = \frac{1}{b} = \text{constant}$, *i.e., the rate of increase of the torque with the time varies inversley as the time.*

This is at once an indication that the two variables, torque and time, are connected by the compound interest law, so that we may write

$$t=a\varepsilon^{b\mathrm{T}}.$$

If $\frac{\theta}{\omega}$ is substituted for t the above equation becomes

$$\theta=a\,\omega\varepsilon^{b\mathrm{T}}.$$

It should be noted that the line in Fig. 4 has been drawn with an equation, $\frac{dt}{d\mathrm{T}}=\frac{t}{30}$, obtained after the value of b had been found from the plottings in Fig. 5.

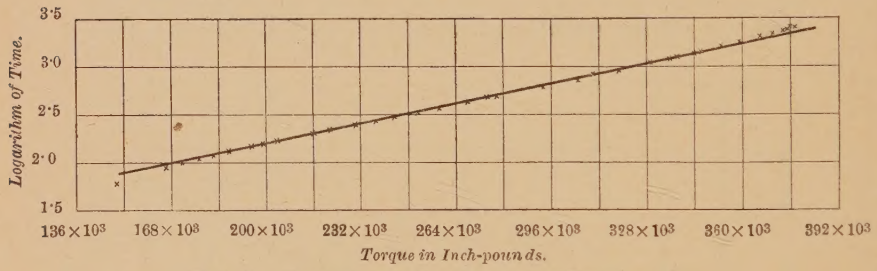


FIG. 5.—TORQUE AND LOGARITHM OF TIME.

Taking the differential equation given above and differentiating we obtain

$$\frac{d^2\mathrm{T}}{dt^2}=-\frac{1}{bt^2}$$

or

$$\frac{d^2\mathrm{T}}{dt^2}\times t^2=\text{constant},$$

i.e., the acceleration which is negative, or, as it may be called, the deceleration of the torque with respect to time varies inversely as the square of the time.

Since $t=a\varepsilon^{b\mathrm{T}}$ represents to a first approximation the relationship between the variables T and t we have

$$\log_{10} t=\log_{10} a+b\mathrm{T}\log_{10}\varepsilon,$$

which is a straight line.

Plotting the experimental values of $\log_{10} t$ and T as in Fig. 5, further confirmation of the truth of the fundamental equation is obtained to at least a first degree of approximation. The

values of the constants a and b are also obtained from these plottings, giving

$$t=8{\cdot}23\varepsilon^{\frac{T}{10}}=8{\cdot}23(1{\cdot}035)^{T},$$

where t is in seconds and T in inch-ton units.

Results similar in form to the foregoing may be stated where θ is the independent variable, since $\theta=\omega t$.

Having deduced the relationships given as representing very closely the results of the experiments, the author wishes to call attention to the plottings of the logarithms of the time near the two axes in Fig. 5. It will be noticed that for a short range of torque the points obtained lie a little below the line drawn suggesting that there may be a small time constant t_0 in the exponential equation.

Making this assumption the equation would then take the form

$$t+t_0=a\varepsilon^{bT},$$

and this equation is the general solution of the differential equation

$$\frac{dt}{dT}=bt+\text{constant},$$

suggesting that the line of Fig. 4 may not strictly pass through the origin although it is close to it. On trial it was found that if t_0 was taken at about 6 seconds and the plottings for Fig. 5 were made between $\log_1$ $(t+6)$ and T the points were brought up to the line given, determining another form of the general equation. This constant t_0 makes no appreciable difference in the value of b.

Owing to the small value of the constant t_0 the other plottings for $\log_{10}t$ are not appreciably affected, the differences in the values of $\log_{10}(t+6)$ as compared with the values of $\log_{10}t$ become negligible within a very short range of the torque. In this connection it should be stated that the duration of twisting was 42 minutes.

The author wishes here to summarise certain other conclusions he has formed as a result of many experiments extending over five years, reserving the full account for later publication.

1. *The Elastic Period.*

(*a*) Plastic strain is always produced by applied stress, and when its amount becomes large enough to be detected by the

strain instrument the actual linearity between stress and elastic strain is no longer directly obvious.

(*b*) The *recorded* limit of elasticity depends on the degree of accuracy to which the strain instrument is capable of indicating the strain, *i.e.*, there is no definite limit of elasticity, the recorded limit being merely an instrument limit.

(*c*) In the case of a tension test where the increase of strain is measured in the direction of the applied stress, linearity between stress and strain is apparently very pronounced, but in the case of a torsion test very small increases in the elongation of helically twisted generating lines of the specimen are accompanied by the relatively large angles of torsion measured by the twist-strain instrument, with the result that plastic strain is observed very early in the test giving low instrument limits of elasticity.*

(*d*) There is elastic strain, whether it is obscured by plastic flow or not, which is closely proportional to the torque (or load) for all values of the torque (or load) up to the maximum, and it is given by $\theta_e/T=\theta_1/T_1$, where T is any torque not exceeding the maximum load, and T_1 and θ_1 are from observations of torque and twist respectively during the elastic period of instrumental linearity between stress and strain. Consequently the usual formula may be used for calculating elastic strains for values of the torque up to near the maximum.†

(*e*) The total strains produced under applied stress, even before the yield period is reached, depend on the time as well as on the torque (or load).

(*f*) The linearity between elastic strain and applied stress may be rendered obvious up to a high value of the load by suppressing the plastic flow.‡

2. *The Yield Period.*

(*a*) The plastic strain phenomena under constant load during this period depends on the time. If the load is imposed on a specimen either *very* slowly or *very* quickly§ there is no yield period, the former case corresponding very closely to isothermal straining and the latter to adibatic.

* *See* Author's Paper, "Engineering," Sept. 3 and 10, 1909; also Author's Paper, "Proceedings" of the Institute of Automobile Engineers, Jan., 1911.

† *See* Paper by the Author, "Minutes of Proceedings" Institute of Civil Engineers, CLXXIX., 1909.

‡ Author's Paper, Institute Civil Engineers, 1909.

§ "Strength of Materials," Ewing, pp. 41-42, 1899 Ed.

(*b*) *Most General Case.*—When the yield torque (or load) is reached and where equilibrium is maintained between the load and the resistance, the velocity of the plastic strain is rapidly and increasingly accelerated for a short period of time due to some rapidly and increasingly softening process going on in the material, the acceleration $\ddot{\theta}$ rapidly reaching a maximum value followed for a short period by a rapidly decreasing acceleration, the velocity $\dot{\theta}$ still increasing until the acceleration reaches zero value with momentary uniform velocity. From this point and for by far the greater part of the yield, the velocity $\dot{\theta}$ undergoes constantly decreasing retardation due to some gradual and increasingly hardening process going on in the material, until the curve in $\ddot{\theta}$ and t becomes, or tends to become, asymptotic to the axis of time with the limit zero deceleration and a uniform value of $\dot{\theta}$.

(*c*) From what has been stated above it follows that the yield period is produced when from the conditions of loading acceleration of plastic flow is produced and further that where the torque-time and torque-twist ratios are kept below certain critical values, and, therefore, where there is no acceleration but only a retardation, no yield period is produced, and the elastic-torque-twist line and the plastic-torque-twist curves are compounded into a smooth and continuous resultant torque-twist diagram.

(*d*) The yield period may be due to the production of a fluid state (liquefaction) in parts of the material under the acceleration with subsequent relegation and hardening producing deceleration after the parts have adjusted themselves to their new positions. The torque-twist curve which is continuous denotes a condition of semi-plastic flow.

(*e*) The yield period which takes place under the circumstances indicated can be raised by strain* and heat treatment at low temperatures to almost any position with respect to a torque-twist curve.

3. *Total Strain.*

(*a*) If the above results are accepted the total strain for any value of the load is given by

$$\underset{\text{total}}{\theta} = \underset{\text{elastic}}{\theta_e} + \underset{\text{plastic}}{\theta_p},$$

* J. Muir: "Transactions" Royal Society, Vol. CXCIII. (1900), and Author's Paper "Minutes of Proceedings" Institute of Civil Engineers, 1909.

Where the elastic strain is a linear function of the load for all ordinary speeds of loading possible with a statical testing machine, and where the plastic strain is function of both load and time.

ABSTRACT.

The author gave an account of the twisting to destruction at a uniform angular velocity of a cylindrical steel specimen 3 in. diameter and of his deductions from the experimental data. The following deductions were made :—

1. The rate of increase of the torque with the time varies inversely as the time.

2. The acceleration of the torque velocity which is negative or, as it may be called, the de-celeration, varies therefore inversely as the square of the time.

3. The variables, time t, and torque T, are connected by the compound interest law.

More exactly $t+t_0=a\varepsilon^{b\mathrm{T}}$, where t_0 is a time constant. Corresponding results in terms of the angle of torsion θ and T obviously followed, since $\theta=\omega t$, where ω is the angular rate of straining.

The author also summarised certain other conclusions he has formed as a result of many experiments extending over five years, illustrating his arguments by means of original diagrams, but reserving the full account for later publication.

DISCUSSION.

Sir R. HADFIELD considered the Paper an important one. With respect to the question of elastic limit, many years ago he was working on manganese steel, which has a very low elastic limit but a very high tenacity, yet it is used for just those purposes for which high elastic steel is used. In carrying out some experiments on manganese steel 25 years ago, by repeated re-toughening the specimen he elongated an 8 in. bar to 14 in. He also drew attention to the behaviour of steel when the rate of stress was exceedingly great, such as in the Admiralty tests with projectiles. A 14 in. shot with 35,000 ft.-tons of energy will pierce a 12 in. plate in $\frac{1}{700}$ second. So we get here problems not met with in ordinary engineering, yet records taken of the passage of the shot show that the steel behaves very much the same under these conditions as it does in an ordinary testing machine.

Dr. C. CHREE pointed out that there were two ways in which the elastic limit could be defined : (1) Either as the range within which Hooke's law holds or (2) as the range within which the strain reduces to zero again when the stress is removed. He thought the results would have been clearer from a physical standpoint if the author had employed t (time) throughout as the independent variable in his mathematical expressions. If he had taken $d\mathrm{T}/dt$ as ordinate and t as abscissa in Fig. 4 his theoretical curve would have been a rectangular hyperbola, which would have served satisfactorily to show the agreement between the observations and the theory. The real physical significance of the experiments seemed to be that under the conditions of test $\theta d\mathrm{T}/dt$ was a constant. Had the author tried whether this relation held when θ did not increase uniformly with the time ? The formula required some restriction as it made $d\mathrm{T}/dt$ initially infinite.

Mr. F. L. HOPWOOD, commenting on Mr. Larard's view that the elastic yield persists during the plastic yield, mentioned that a similar conclusion had been drawn by Andrade (" Proc." Roy. Soc., A., Vol. 84, 1910) from tension experiments on lead and copper.

Prof. MARGETSON pointed out that since the rate of strain was constant, Figs. 2 and 3 were practically similar, and he suggested that if the results had been stated in terms of θ and T, instead of between t and T, the results would have been more in accordance with the author's previous statement * of the compound interest law in the form $ae^{bT}=\theta$.

Mr. R. APPLEYARD, in drawing attention to Fig. 5, remarked:—There is a certain amount of divergence between the calculated firm line and the line passing through the plottings, and Mr. Larard suggests that to account for this it may be necessary to introduce a small time-constant t_0. It should be observed, however, that in working with logarithmic curves a small error in the constants is, as a rule, sufficient to produce a divergence of the kind here obtained. Hence it is worth while considering whether the divergence is due to the author having selected scarcely the best value for b. In Fig. 4 the observation points are somewhat scattered, and although it is convenient to draw a line from the origin to the point (2400, 80) which makes $b=\frac{1}{30}$, this is not necessarily the best line to fit the final result of Fig. 5. If the line of dots in Fig. 5 is continued backwards to cut a vertical line through the origin it will be found to give a value

$$\log_{10} a=0{\cdot}64.$$

Hence, $a=4{\cdot}37$, instead of the author's 8·23, would meet this case. It is, therefore, desirable to examine b very carefully before assuming the existence of a quantity t_0. The relation between T and t is, therefore, in this case

$$\log_{10} t=0{\cdot}64+b\mathrm{T}\log_{10} e.$$

Now, take the best point in Fig. 5, and it is seen that there is agreement, where $\mathrm{T}=264\times10^3$, and $\log_{10} t=2{\cdot}625$, as nearly as can be judged from the diagram. Hence, b can be derived from

$$2{\cdot}625=0{\cdot}64+\frac{b\times264\times10^3\times0{\cdot}4343}{2{,}240},$$

since T has here to be in inch-tons. This gives $b=0{\cdot}0388$, or $\frac{1}{b}=25{\cdot}8$. Now in Fig. 4 is co-ordinated $\frac{dt}{d\mathrm{T}}$ and t, and since

$$\frac{dt}{d\mathrm{T}}=bt,$$

we can select a value, say, $t=2{,}400$, and we find

$$\frac{dt}{d\mathrm{T}}=0{\cdot}0388\times2{,}400=93{\cdot}07,$$

which is well above the 80 selected by the author. This point, if joined to the origin, gives a line which is within the observation points, and the effect on Fig. 5 would be that the firm line in that diagram would pass through the plottings without the introduction of a new factor t_0.

The AUTHOR replied, and supplemented his remarks as follows: The shot experiment quoted by Sir Robert Hadfield is suggestive of problems of great importance from a physical as well as from a practical point of view. If it be established as true in a more universal application that the stress-time rate multiplied by the strain is constant then it follows that if the velocity and energy of impact are great enough the small fraction of a second representing the duration of impact is insufficient to allow of the development of disintegrating strain in the bulk of the shot, and so penetration takes place. The Author agrees with Dr. Chree that the real physical significance of the

* "Proceedings" Institution of Mechanical Engineers, 1907.

experiment is that $\theta\frac{dT}{dt}$ is constant. This interpretation was definitely stated by him in the circulated abstract of his British Association Paper.* In further reply to Mr. Appleyard the author states that he has not made the mistake of allowing logarithmic differences to involve him in any serious error. The line given by him in Fig. 5 gives an intercept which is the log of 8·23. That the value $b=\frac{1}{30}$ is evidenced by the fact that when the different values of dT/dt are multiplied by the *measured* angles of twist the products, throughout the entire range from the end of the yield to near the maximum load are astonishingly close to 3·435, this constant being derived as follows :—

Since
$$\frac{dT}{dt}=\frac{1}{bt}-\frac{\omega}{b\theta},$$

we get, on substituting the values $\omega=0{\cdot}1145$ and $b=\frac{1}{30}$

$$\theta\frac{dT}{dt}=3{\cdot}435.$$

Mr. Appleyard in drawing another line for Fig. 5 may not perhaps know that in all such diagrams there is a slight flexure downwards near the bottom of the line and a slight flexure upwards at the other extremity. The author is investigating the significance of this peculiarity. With respect to the interpretation of the constant t_0 first suggested, but afterwards qualified, the author has some feeling of uncertainty. It is not unlikely that its place should be taken by a very slowly increasing function of the torque corresponding to the *growth* of the elastic strain with the time. Replying further to Mr. Hopwood, the author holds the view, as may be gathered by this Paper and his previously published Paper,† that for steel not only does the elastic strain persist, but that it is continuously augmented under increasing load and time until it reaches its maximum value at some load not far below the maximum.

In reply to Mr. Hopwood the Author stated that if his results on "double and simultaneous flow" had been in any way anticipated, so much the better for the theory, but at the same time he called attention to the results in his previous Papers. Prof. Margetson's point was covered in the Paper referred to since the angular velocity of straining was there given.

* "Report," British Association, 1912.
† "Proceedings" Institution of Civil Engineers, 1909.

IX. *The Effects of Holes and Semicircular Notches on the Distribution of Stress in Tension Members.* By E. G. Coker, *M.A., D.Sc., Professor of Mechanical Engineering in the City and Guilds of London Technical College, Finsbury.*

Received and Read November 10, 1911.
Received in Revised Form November 12, 1912.

In a previous Paper* the author has determined approximately the difference of the principal stresses at the minimum sections of tension members having notches of various forms, and in the present Paper some experimental values are obtained for circular holes and semicircular notches in tension members, and these are compared with the results of calculations. The method of experiment adopted is similar to that described in a former Paper.† A strip of transparent xylonite is cut into shape and subjected to stress in any convenient way. The distribution of the stress is then examined by the optical effects produced when plane or circularly polarised light is passed through the plate at a perpendicular incidence.

The optical effect of the stress distribution at any point is to cause a retardation, R, in the component rays into which the incident beam is broken up, and to a first approximation R is proportional to the difference of the principal stresses $\widehat{\theta\theta}$ and $\widehat{rr}$ at a point r, θ, and to the thickness T of the plate, so that we have

$$R=(\widehat{\theta\theta}-\widehat{rr})CT$$

where C is a constant.

The retardation can be measured directly in terms of a wave-length scale, and the stress difference can be inferred therefrom, but in the present Paper the numerical values have in all cases been determined by reducing the field of view to darkness by using a stressed member placed along the direction of one of the principal axes of stress at the point considered, so that the stress difference is measured directly.

A case of primary importance is that afforded by a cylindrical hole in a tension member, such as occurs in many forms

* "Trans." Inst. Naval Architects, April, 1911.

† "The Optical Determination of Stress," by E. G. Coker, "Phil. Mag.," 1910.

of engineering construction, although generally the conditions are complicated by the stress caused by the pressure of a rivet in the hole, and also by the stress due to the grip of the rivet heads. For plane stress the equations of equilibrium are satisfied when the radial stress $\widehat{rr}$, the tangential stress $\widehat{\theta\theta}$, and the shear stress $\widehat{r\theta}$ can be expressed in terms of Airy's function F, where

$$\widehat{rr}=\frac{1}{r^2}\frac{\partial^2 F}{\partial\theta^2}+\frac{1}{r}\frac{\partial F}{\partial r},\quad \widehat{\theta\theta}=\frac{\partial^2 F}{\partial r^2},\quad \widehat{r\theta}=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial\theta}\right) \quad . \quad . \quad (1)$$

For the case of a hole in an infinite plate subjected to stress p it can be shown* that the function F has the value

$$F=\tfrac{1}{4}p\left\{r^2-2a^2\log r-\frac{(r^2-a^2)^2}{r^2}\cos 2\theta\right\} \quad . \quad . \quad . \quad (2)$$

from which we readily obtain the stresses in the form

$$\widehat{rr}=\tfrac{1}{2}p\left\{1-\frac{a^2}{r^2}+\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right\} \quad . \quad . \quad . \quad (3)$$

$$\widehat{\theta\theta}=\tfrac{1}{2}p\left\{1+\frac{a^2}{r^2}-\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right\} \quad . \quad . \quad . \quad . \quad (4)$$

$$\widehat{r\theta}=\tfrac{1}{2}p\left\{-1-\frac{2a^2}{r^2}+\frac{3a^4}{r^4}\right\}\sin 2\theta, \quad . \quad . \quad . \quad . \quad (5)$$

where a is the radius of the hole.

The same result has been recently obtained by Suyehiro† using the method of generalised plane stress. For a tension member with a small hole in it sufficiently far from the edge we see that the stress across the minimum transverse section rises slowly as we approach the discontinuity, and finally very rapidly. When $r=a$ and $\theta=\frac{\pi}{2}$, we have $\widehat{\theta\theta}=3p$ and $\widehat{rr}=\widehat{r\theta}=0$, or the maximum stress is three times the uniform stress at a distance from the discontinuity.

In order to experimentally determine the stress at the minimum section of a tension member when pierced by a central hole, strips of xylonite were prepared, 1 in. in width and 0·186 in. thick, and holes were cut in these ranging from $\frac{1}{8}$ in. to $\frac{1}{2}$ in. These specimens were stressed in a simple form of apparatus for applying definite tension loads.

* Föppl, "Vorlesungen über Technische Mechanik," Vol. V., p. 352.

† "Engineering," September 1, 1911.

In the earlier experiments the holes were drilled through from one side of the plate in the ordinary fashion, but this was found to slightly damage the material, owing to the drill pushing the last section of uncut material outwards and causing a slight strain around the edge of the hole. In order to overcome this source of error, which had a considerable influence on the stress determinations, the holes were drilled from both sides, and rather smaller than the final size, the correct diameters being obtained by removing the remaining material with a fine file. This treatment caused no injury to the specimen and more accurate stress determinations became possible.

At first a total load of 200 lb. was applied to the specimens, but this was afterwards reduced to 100 lb. to minimise possible errors due to imperfect elasticity of the material at high loads.

As a result of an optical examination of the transverse sections through the centres of five holes of different diameters the following results (Table I.) were obtained of the differences of the principal stresses at various points in the section.

TABLE I.

Total load 100 lb. Section of specimen, 1 in. × 0·186 in.

Experiment No.	Size of hole. In.	Measured difference of the principal stresses in pounds per square inch at a distance (in inches) from the edge of—								p_m mean stress intensity, lb./sq. in.	p.
		0	$\frac{1}{8}$	$\frac{2}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{15}{32}$		
1	$\frac{1}{16}$	545	550	...	560	...	570	615	1,470	584	549
2	$\frac{1}{8}$	560	575	...	595	...	622	1,560	...	620	547
3	$\frac{1}{4}$	580	585	...	640	760	1,770	...	...	724	568
4	$\frac{3}{8}$	625	630	...	890	1,850	...	...	...	868	570
5	$\frac{1}{2}$	635	760	1,070	2,040	...	...	...	...	1,035	613

Some of these values are plotted in Fig. 1, and a comparison of these numbers with the values obtained by calculation shows that, when the hole is small compared with the breadth of the plate, there is a very close agreement.

It may be remarked that the maximum values are difficult to obtain, as they are point values at the periphery of the hole, where the variation of stress in a radial transverse section is very great, and determinations are therefore more liable to error here than at any other point.

In order to determine the law of variation of stress along the cross-section the stresses were calculated for the ideal case of a

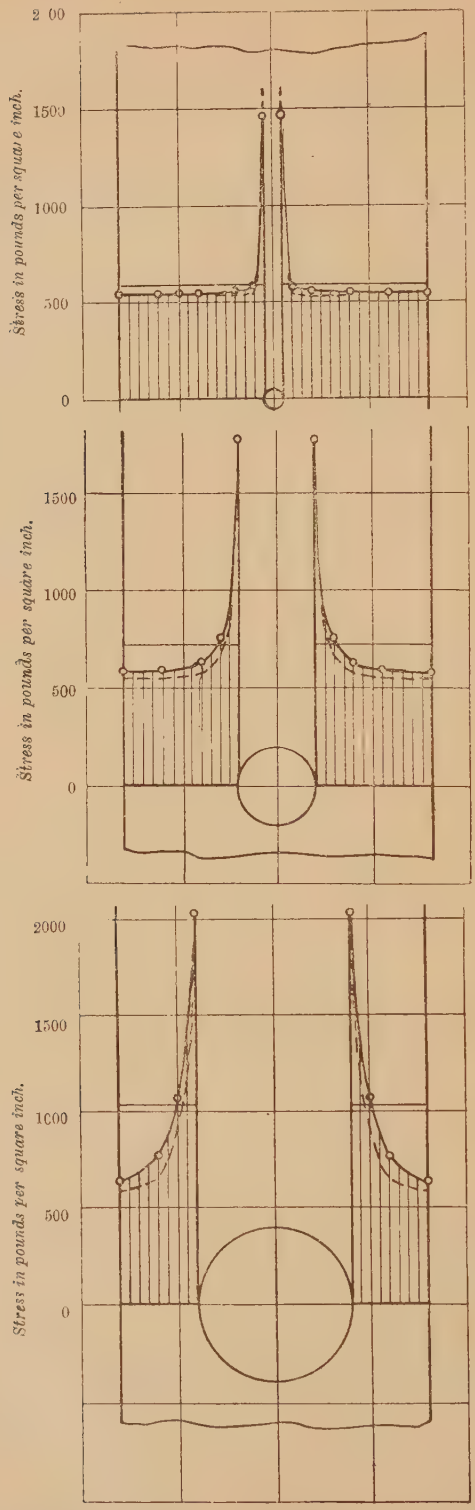


FIG. 1.—STRESS DISTRIBUTION IN A TENSION MEMBER WITH A CENTRAL HOLE. NOS. 1, 3 AND 5 OF TABLE I. (The calculated values are shown by dotted lines.)

hole in an infinite plate subjected to uniform tension, and these were compared with the values obtained in Table I. The principal stresses $\widehat{\theta\theta}$, $\widehat{rr}$ at the cross-section are given by

$$\widehat{\theta\theta}=\frac{p}{2}\left(2+\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\widehat{rr}=\frac{3}{2}p\left(\frac{a^2}{r^2}-\frac{a^4}{r^4}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and from these we obtain the following ratios for values of r/a between 1 and 10.

TABLE II.

r/a	1·0	1·25	1·5	2·0	3·0	4·0	5·0	10·0	∞
$\widehat{\theta\theta}/p$	3	1·934	1·519	1·219	1·074	1·050	1·022	1·005	1
$\widehat{rr}/p$.....	0	0·346	0·370	0·281	0·148	0·090	0·058	0·015	0
$\widehat{\theta\theta}-\widehat{rr}/p$.....	3	1·588	1·149	0·938	0·926	0·960	0·964	0·990	1

The experimental determinations differ from the ideal case in that the uniform stress p, corresponding to a plate of infinite width, is not directly ascertainable, and in order to obtain its value for any case we may select two or more experimental values and compare them with the coefficients in Table II. to obtain a value of p. If this is done it will be found that the curves of stress variation agree very well with the experimental values, but this is not a very accurate method, and a more searching test is to determine the value of p from the load applied to the plate, assuming that the principal stress variation $\widehat{\theta\theta}$ follows the law given by equation (6). If the mean stress across the section is p_m and the edge of the plate is at a distance $c \cdot a$ from the centre we have

$$p_m c(a-1)=\int_a^{ca}\widehat{\theta\theta}\,.\,dr=\int_a^{ca}p\left(1+\tfrac{1}{2}\frac{a^2}{r^2}+\frac{3}{2}\frac{a^4}{r^4}\right)dr,$$

from which we readily obtain a formula for calculating p in terms of p_m and the dimensional value c in the form

$$p_m=p\left(1+\frac{1}{c}+\frac{1}{2c^2}+\frac{1}{2c^3}\right). \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The values of p, for each case, have been determined from this latter equation, and the calculated stresses, corresponding to the observations, are plotted for comparison with the observed values, and are shown by dotted curves in each figure.

As will be seen from Fig. 1, there is a very fair agreement for small holes and a less good agreement for large ones, but these latter observations lie outside the limits contemplated by the mathematical solution. It appears that, having established having a frustum was greater than one quarter of the width of the plate at hand, we may assume that a set of reflexions, a series of equidistant without serious error.

If the plate under [illegible] assumption, which may [illegible] be considered from the applied load by the formula

$$\text{[illegible]} \quad \ldots \quad (8),$$

and it is agreed compared with unity the values in the original approximate formula

$$\text{[illegible]} \quad \ldots \quad (10).$$

If the hole has a greater diameter than one quarter of the width of the plate, the approximate theory does not [illegible] shows a greater [illegible] calculated by these approximate formulæ.

Another case of importance is that of a transverse crack, or a [illegible] notch.

A perfectly general solution has been obtained by [illegible] and an approximate solution is given by [illegible] for a crack symmetrically [illegible] Assuming [illegible] that the radial and tangential stresses in the centre of the semicircle are expressed by

$$\text{[illegible]} \quad \ldots \quad (11),$$

we obtain

$$\text{[illegible]} \quad \ldots \quad (12),$$

which satisfy the body stress equations, but not all the surface conditions. [illegible]

* "On an Equation of a Common Kind in Optics and Mechanics," *Proc. Roy. Soc.*, Vol. [illegible], pp. [illegible].

** "Centenarbeiche Wechselwirkung und den Theoretischen Elastizitäts," February, 1905.

the experimental values, and they indicate that the stress is doubled at the notch, for if $r=a$, we have

$$\widehat{rr}=0, \quad \widehat{\theta\theta}=2p(1-\sin^2\theta), \quad \widehat{r\theta}=p\sin\theta\,.\cos\theta,$$

so that when $\theta=0$, $\widehat{\theta\theta}=2p$, and this is the maximum value obtained.

The general values of the principal stresses at the section passing through the centre of the notch and perpendicular to the direction of the stress are

$$\widehat{\theta\theta}=\frac{p}{2}\left(\frac{a^4}{r^4}+\frac{a^2}{r^2}+2\right), \quad \ldots\ldots \quad (13)$$

$$\widehat{rr}=\frac{p}{2}\left(\frac{a^2}{r^2}-\frac{a^4}{r^4}\right). \quad \ldots\ldots \quad (14)$$

and for purposes of comparison with the results of experiment the following table of coefficients is given :—

TABLE III.

r/a	1	1·25	1·5	2·0	3·0	4·0	5·0	10·0	∞
$\widehat{\theta\theta}\ p$	2	1·525	1·321	1·156	1·062	1·033	1·021	1·005	1
$\widehat{rr}\ p$.....	0	0·115	0·123	0·094	0·049	0·029	0·019	0·0049	0
$(\widehat{\theta\theta}-\widehat{rr})\ p$ ...	2	1·410	1·198	1·062	1·013	1·004	1·002	1·0001	1

An optical examination was made of specimens having notches symmetrically arranged with respect to a transverse section as the most convenient form, and determinations of the differences of the principal stresses are shown in the accompanying Table IV., and some of these values are plotted in Fig. 2.

The solution given by equation (12) implies infinite width, but since finite width is a practical necessity it seems best on general grounds to take for experimental comparison the symmetrical form used here.

TABLE IV.

Experiment No.	Radius of notch. In.	Measured differences of principal stresses in pounds per square inch at a distance (inches) from the centre line of— 0	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{15}{32}$	Mean stress intensity, lb./sq. in. p_m.	p.
1	$\frac{1}{32}$	515	515	...	515	...	470	500	1,070	584	527
							670				
2	$\frac{1}{16}$	530	530	...	525	...	490	1,160	...	620	571
3	$\frac{1}{8}$	535	535	...	515	705	1,250	...	...	724	616
4	$\frac{3}{16}$	515	545	...	740	1,340	...	...	...	868	680
5	$\frac{1}{4}$	560	685	890	1,660	...	...	...	...	1,035	742

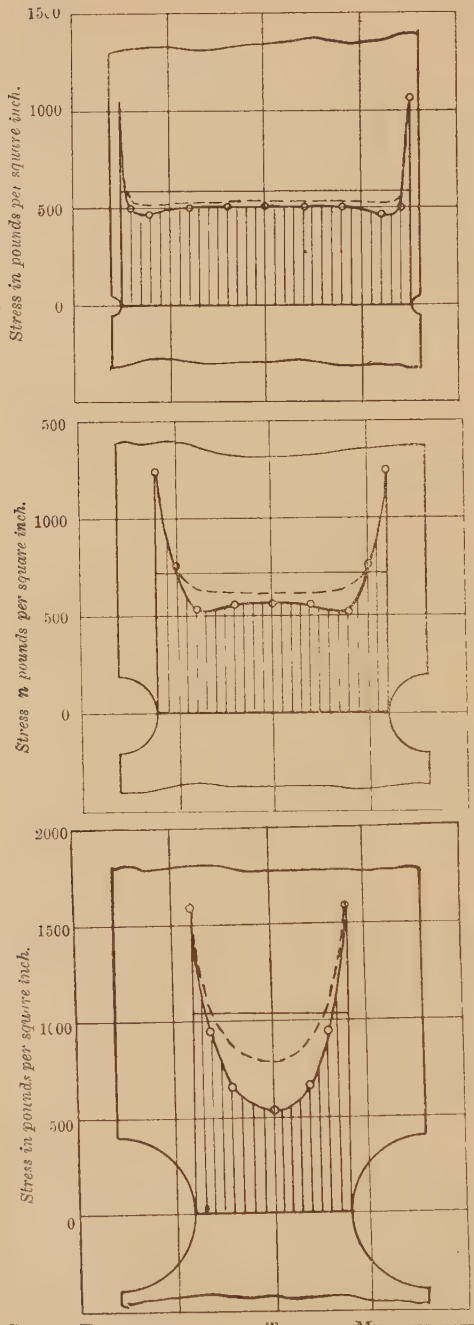


FIG. 2.—STRESS DISTRIBUTION IN A TENSION MEMBER WITH SEMICIRCULAR NOTCHES. NOS 1, 3 AND 5, TABLE IV. (The calculated values are shown by dotted lines.)

To determine how far the variation of stress agrees with the approximate expression obtained above the value of p is found from the equation

$$p_m a(c-1)=\int_a^{ca}\widehat{\theta\theta}\,.\,dr=\frac{p}{2}\int_a^{ca}\left(2+\frac{a^2}{r^2}+\frac{a^4}{r^4}\right)dr,$$

where ca is the half breadth of the plate from which we obtain the expression

$$p_m=p\left(1+\frac{2}{3}\frac{1}{c}+\frac{1}{6}\frac{1}{c^2}+\frac{1}{6}\cdot\frac{1}{c^3}\right). \quad . \quad . \quad . \quad (15)$$

The values of the stress variation obtained from this value of p are obtained from the last line of Table III., and are plotted for comparative purposes in Fig. 2. The agreement for small notches is fairly close, but, for those cases which clearly lie beyond the conditions assumed by the mathe matical analysis, the results indicate that probably both principal stresses differ rather markedly from the calculated values. A want of agreement is also noticeable for notches of $\frac{1}{32}$ in., $\frac{1}{16}$ in. and $\frac{1}{8}$ in. radius in the neighbourhood of $r/a=2$, where the minimum values are reached, and as the radial stress rises to its maximum near this value it seems probable that the radial stress equation is not so close an approximation as the value for the tension across the section. For the larger notches the formula does not agree very well with the experimental values except at the maxima.

It is, therefore, not justifiable to state any general rule for the maximum stress due to a notch which has a radius greater than one-eighth of the breadth of the member, but within this limit we may express the maximum stress in terms of p_m and c in the form

$$p_{\text{max.}}=p_{\text{(mean)}}\cdot\frac{12c^3}{6c^3+4c^2+c+1} \quad . \quad . \quad . \quad (16)$$

and if c is comparatively large so that the first power may be neglected, we have

$$p_{\text{max.}}=\frac{2c}{c+\frac{2}{3}}p_{\text{mean}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

as an approximate expression for use in calculating the greatest stress produced in a tension member with symmetrical notches of semicircular form.

A preliminary step to the solution of the second problem

has been made by L. F. Richardson,* and his tentative results differ somewhat from the experimental values here found.

It is hoped that it will be possible to undertake a joint investigation by a method suggested in a former Paper † to compare with the results already obtained.

In conclusion the author wishes to thank Dr. Chree for kindly reading and criticising the manuscript.

ABSTRACT.

The necessities of practical construction lead to a number of interesting cases of stress problems in which discontinuities like holes and notches occur in great variety both as regards form and arrangement.

For the experimental determination of stresses in loaded members an optical examination of a model shaped in transparent material has many advantages.

Two cases of primary importance are examined in this way, and the results are compared with those obtained by analysis. The first example relates to the case of a hole in a tension member subjected to a uniformly applied stress, p.

The values of $(p_x - p_y)$ the difference between the principal stresses are readily obtained optically, and they show a fair agreement with the calculated values if the diameter of the hole is not greater than one-quarter of the width of the plate, but beyond this the agreement is not so good. For practical purposes it is important to be able to estimate the maximum stress from the value obtained by assuming that the total load on a tension member is uniformly distributed over the cross-section. A formula based on the relationship found in the experiments takes the form

$$p_{\max.} = \frac{6c^3}{2c^3+2c^2+c+1} p_{\text{mean}},$$

where c is the ratio of the width of the member to the diameter of the hole; if c is large compared with unity this reduces to the simple form

$$p_{\text{mean}} = \frac{3c}{c+1}.$$

In the case of two semicircular notches, arranged symmetrically with regard to the centre line and to the cross-section, there appears to be no exact mathematical solution, but an approximate one has been obtained by Leon, resulting in expressions for p_x and p_y at the minimum section of the form

$$p_x = \frac{p}{2}\left(2 + \frac{a^2}{r^2} + \frac{a^4}{r^4}\right), \quad p_y = \frac{p}{2}\left(\frac{a^2}{r^2} - \frac{a^4}{r^4}\right),$$

provided that the radius of the notch is small compared with the breadth of the plate.

* "The Approximate Solution of Various Boundary Problems by Surface Integration combined with Freehand Graphs," by L. F. Richardson, "Proc." Physical Society, February, 1911.

† "The Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam," by L. F. Richardson, "Phil. Trans.," 1910.

Experimental determinations of $p_x - p_y$ show that the maximum values agree very well with those of the formulæ for notches having a maximum radius of about one-quarter of the breadth of the member, but the minimum values do not show a very good agreement if the notch has a radius greater than one-eighth of the breadth. The results appear to indicate that the radial stress for large notches is greater than that given by the formula.

For determining the maximum stress from the applied mean stress a formula is proposed of the form

$$p_{\max.} = \frac{12c^3}{6c^2 + 4c^2 + c + 1} p_{\text{mean}},$$

and this shows a fair agreement with the experimental values.

X. *A Column Testing Machine.* *By* E. G. Coker, *M.A., D.Sc., Professor of Mechanical Engineering in the City and Guilds of London Technical College, Finsbury.*

Received November 18, 1912. Read November 22, 1912.

Experimental determinations of the strength of long members under compression loads are of considerable importance, as, in practice, members of this kind are nearly as numerous as those subjected to tension, but the results available are not nearly so complete as for tension members since compression tests are more difficult to carry out, and machines, which accurately fulfil the conditions required for experiment, are not numerous and are, moreover, very costly. It is essential in compression testing machines to ensure that the conditions of end fixing are realised, and that the load can be accurately measured. Both requirements are more difficult to carry out for compression than for tension, as a pillar under load is always tending towards an unstable condition, while a tension member acts in the reverse manner. The required conditions of end fixing can be obtained if the end plates applying the load are maintained perpendicular to the axis of the originally straight pillar, and in some small machines the end bearing plates move on slides to ensure parallelism, an obviously defective arrangement, since the friction of the slides renders it difficult to measure the applied load.

In some very large machines, for full-sized pillars, a massive head is secured by tension rods to a hydraulic cylinder, and the ram of this latter applies the load, usually in a very accurately axial direction. For rough measurements the total load is determined from the hydrostatic pressure, and is subject to a deduction for the friction of the cup leathers, usually a large correction of uncertain amount. This can, however, be very much reduced if the cup leathers are dispensed with and the ram or plunger made a very true fit in the cylinder. The plunger then simply floats in oil, which leaks slowly between it and the cylinder wall, and the friction is very small and becomes practically negligible if the ram is rotated.

In some other forms the load, applied by a hydraulic plunger of the usual type, is weighed by aid of a large circular diaphragm carrying one end plate and clamped at its edges in a casing to enclose a thin layer of liquid. The pressure registered by the

imprisoned liquid enables the load to be weighed with great accuracy, while the arrangement also permits the bearing plates to remain perpendicular to the axis of the specimen. In the well-known Emery testing machine this pressure is weighed on a smaller diaphragm by a subsidiary system of levers.

A mechanical weighing device attached directly to an end-pressure plate usually necessitates this plate having more than one degree of freedom, a bad defect in a testing machine of this kind. This difficulty may, however, be overcome, and the simple machine described here uses a lever system to measure

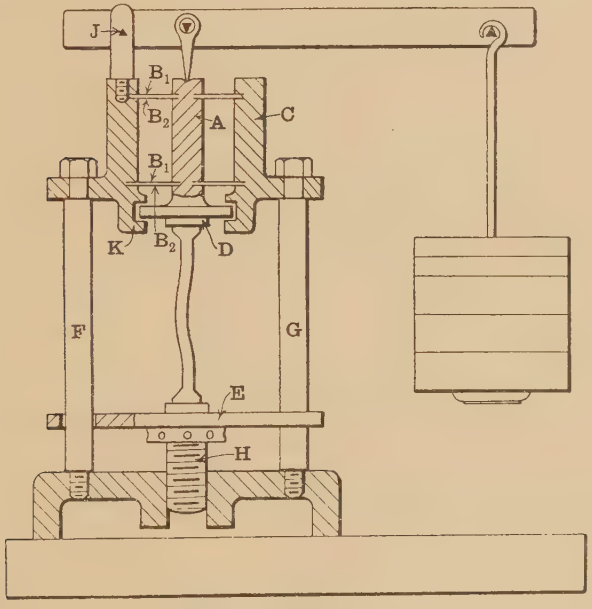


FIG. 1.

the load on a plunger, which is constrained to move axially without appreciable friction. This is accomplished by controlling its motion by aid of thin spring plates or wires, which are secured to the movable plunger and to a fixed casing, so that all degrees of freedom are suppressed except one. In one arrangement, which has been designed for testing small models of pillars made of transparent materials, the central plunger is constructed to grip the inner edges of annular discs placed with their planes a considerable distance apart and clamped to a fixed casing.

A diagram of this arrangement is shown in vertical section. Fig. 1, in which the plunger A is secured at each end to a pair of very thin brass or steel diaphragm plates, B_1, B_2, which latter are clamped at their outer edges in a casing, C, of annular cross-section. The plunger so secured has a small amount of vertical motion only, and the pillar to be tested is set between the compression plates D and E, the latter of which slides on the guide bars F and G, and is adjusted in height by a flat-topped screw, H, secured in the base.

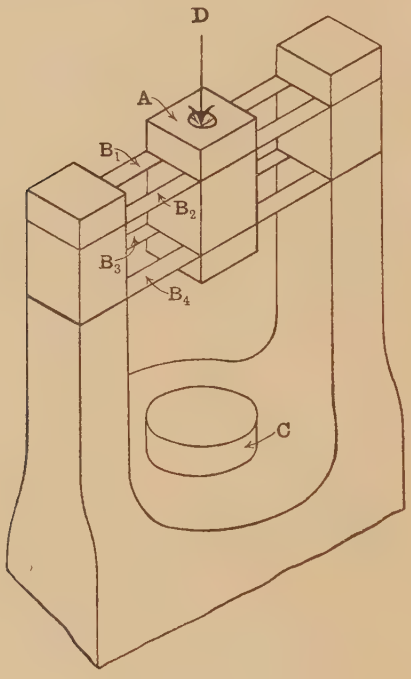


Fig. 2.

In large machines a hydraulic ram replaces the screw, and the sliding plate E is not required. The load borne by the plunger is weighed, in this example, by a single lever pivoted at J and carrying a weight at its outer end. The lever is maintained in a horizontal position by the screw, and the plunger is limited to a very small amount of motion by stops K. An index finger, not shown, is also provided to show when the plunger is freely supported by the diaphragms.

Photographs of the stress distribution show that this apparatus gives satisfactory results for transparent pillars under usual conditions of end fixture.

In some forms of apparatus it may be more convenient to use perforated diaphragms, or thin wires arranged radially in a plane. In others it may be more convenient to set thin rectangular strips in parallel planes, and the model of an arrangement for a small machine, Fig. 2, shows a block, A, supported by four steel strips B_1, B_2, B_3, B_4, arranged so that all degrees of freedom except one are suppressed. In this arrangement the load is applied by the plunger C of a hydraulic cylinder moving in the same line as the block A, and the load transmitted through the pillar to this block can be measured at the point D by a weighted lever system, or other suitable arrangement.

The application of thin plates, wires and diaphragms to give one degree of freedom to a pressure enables the load to be measured with great accuracy, since there is no appreciable loss due to friction, and the energy stored in the elastic guiding members, when these latter are displaced from their central configurations, is restored when the equilibrium positions are again attained.

In conclusion, the Author desires to express his thanks to his assistant, Mr. F. H. Withycomb, who has been very helpful in the detailed design and construction of the apparatus.

ABSTRACT.

The conditions of fixture of the ends of columns, and the large influence this has upon their strength, generally make it necessary to use special testing machines for these members, in which the end plates applying the load are accurately parallel, and remain so during a test.

If only rough measurements of the load are required this offers no serious difficulty, but accurate measurement involves elaborate mechanical devices, some of which are briefly referred to in the Paper.

This difficulty is overcome in a simple manner by supporting one pressure plate by two or more annular diaphragms spaced at considerable intervals, and clamped at their outer edges to a fixed casing in such a manner that only one degree of freedom is possible This construction is carried out in the machine described in the Paper. The total load on the pressure plate supported in this way can be measured by a loaded lever system, or other sutable means. The other pressure plate may be carried on guides, and the load applied by screws, hydraulic pressure or other suitable means.

Rectangular plates or wires may also be used in place of annular diaphragms, and a model showing an application to a compression machine is described.

Photographs of celluloid columns are shown under stress, and the colours produced by temporary double refraction indicate that the loads are satisfactorily applied in a machine of this type.

DISCUSSION.

Sir R. HADFIELD thought this Paper opened out an important field of research.

Mr. C. E. LARARD asked if the machine was meant for testing metals or only xylonite, as in the former case the conical pivot at the top would be a source of weakness.

The AUTHOR replied that for heavier machines the conical pivot could be replaced by a narrow strip of thin steel plate which he considered preferable to a knife edge.

XI. *The Electrical Conductivity and Fluidity of Strong Solutions.* *By* W. S. Tucker, *B.Sc.*

Received November 22, 1912. Read January 24, 1913.

It may be fairly assumed that when solutions are of such concentration that the molecules of dissolved substance occupy a relatively large volume, the ions produced will interfere with each other's movements. If the ionisation factor be calculated from the ratio of the molecular conductivity to that at infinite dilution, we get a result which fails to take into account this interference, and therefore cannot be regarded as reliable.

In Prof. Callendar's association theory of strong solutions* the depression of freezing point and elevation of boiling point can be foretold by associating with each molecule and ion a certain constant number of water molecules. With the data at his disposal he obtains good agreement between the experimental values and his calculated values for cane sugar up to any concentration, and for solutions of potassium, sodium and magnesium chlorides. For the strong solutions of calcium chloride discrepancy occurs, most probably owing to unsatisfactory ionisation data.

The experiments here described were made to find whether there is any relation between conductivity and fluidity.

Certain experiments have already been performed by Lyle and Hosking † with sodium chloride, and later by Hosking‡ with lithium chloride, but they did not work with so many strong solutions, nor did they cover the same ranges of temperature.

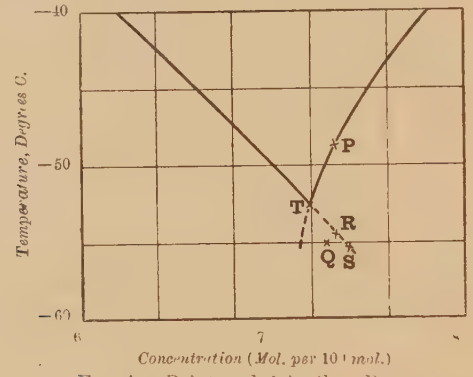
Calcium chloride solution, with its low cryohydrate point and its extreme solubility, forms a very good solution to work on. Moreover, it can be highly supercooled. This can be illustrated by taking a solution slightly above cryohydrate strength and cooling it.

Referring to the curves shown in Fig. 1, crystals of the salt $CaCl_2 6H_2O$ should separate out at P, but the solution supercools to Q. The addition of a fragment of ice causes rise to R, giving the approximate freezing point. Ice now separates

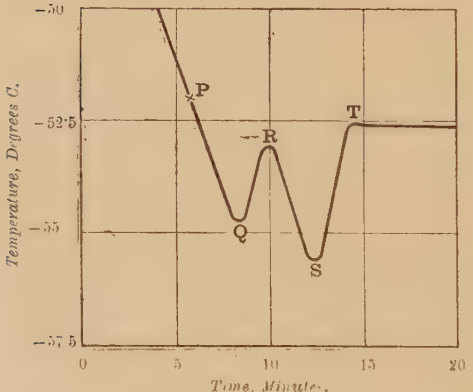
* Callendar, "Proc." Roy. Soc., A, Vol. 80, 1908, p. 466.
† Lyle and Hosking, "Phil. Mag.," May, 1902, p. 487.
‡ Hosking, "Phil. Mag.," May, 1904, p. 469.

from R to S. At S fresh crystals presumably of the salt make their appearance, causing a rise to T, probably above cryohydric point; but separation of these crystals quickly brings the solution to the constant temperature of the cryohydric point.

A supercooling of 15°C. can be obtained with proper precautions, when the crystals are dissolved in their own water of crystallisation.



Freezing Point and Solubility Diagram.



Cooling Curves.

FIG. 1.

The feature of the method here described is the simultaneous observation of temperature, viscosity and resistance. The electrolytic cell and viscometer are immersed in the solution for which they are employed, and both are bound to the bulb of a platinum thermometer, which serves also as a stirrer.

Measurement of Conductivity.

The electrolytic cell C is a flattened tube drawn out into a flat jet. It contains a platinised-platinum electrode, P, whose dimensions are 25 mm. by 5 mm. The second electrode, also of platinised-platinum, forms a cylindrical sheath, Q, round the bulb of the thermometer T. (*See* Fig. 2.)

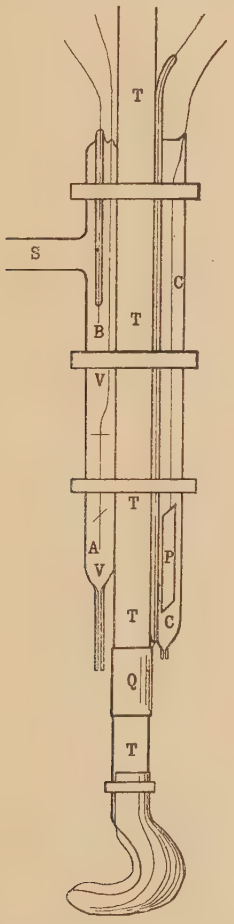


FIG. 2.—APPARATUS FOR DETERMINING VISCOSITY AND CONDUCTIVITY.

The thermometer is mounted on a frame to which a vertical oscillatory motion is imparted. This motion is caused by a connecting rod attached to a large pulley driven by a motor.

As the thermometer oscillates it draws through the liquid a platinum scoop, which acts as an efficient stirring arrangement.

The resistance was measured by that modification of Kohlrausch's method described by Fitzpatrick,* in which a high-speed double commutator and a moving-coil galvanometer replace the coil and telephone. An accuracy of 1 part in 2,000 could be thus obtained, and was greater than that possible with the viscosity apparatus.

A preliminary experiment was made which showed that the oscillatory motion of the cell and the consequent motion of the liquid in it did not affect the resistance readings. Such oscillatory motion ensured the temperature of the liquid in the jet being exactly that of the liquid in the bath.

In the observations taken the concentration was expressed in gram-molecules per 100 molecules of water. This concentration remains constant with change of temperature, and was employed by Prof. Callendar in his Paper on strong solutions.

The capacity of the cell was found by the use of a standard solution employing Kohlrausch's results.

The molecular weight of the anhydrous salt is taken as $110{\cdot}9_2$.

Measurement of Viscosity.

The viscosity is measured by a modification of Poiseuille's method. The tube through which the flow occurs is a uniform jet about 2 cm. long drawn out from a piece of quill tubing, V, as shown by the diagram, Fig. 2. Connection can be made by means of the side-tube S with an aspirator or with a compressed air reservoir. When the tube is immersed in the solution, the latter can be drawn in or driven out at will.

The viscometer is plunged into the solution to a certain depth, and the time of inflow is measured and compared with that of water under the same conditions.

During the experiments the viscometer is invisible, hence the levels of the liquid when flowing into the tube are indicated by electrical means.

Two fine platinum wires A and B serve as contacts and are insulated from one another. As the liquid rises within the tube it makes circuit with Q and A through the liquid and the motion of a galvanometer is noted. A second motion is pro-

* B.A. "Report," 1886, p. 328.

duced when contact with B is made. Thus the passing of the levels can be accurately timed.

The assumption is now made that the rate of influx of the solution is proportional to the hydrostatic pressure, and inversely proportional to the viscosity.

The accuracy of this assumption was checked by an experiment with pure water, whose viscosity has been accurately found by Thorpe and Rodger.*

Two experiments are performed.

1. The water is run into the tube under its own pressure, the initial and final differences in level being H and h_1 respectively. It can be shown that

$$\frac{H}{h_1}=e^{\frac{K}{a}g.\frac{\rho}{\eta}t_1}=e^{mt_1},$$

where ρ is the density, η the viscosity, t the time of inflow, K a constant depending on the dimensions of the jet, and a the area of the wider portion of the tube.

2. An external pressure P is applied by means of the aspirator, and this acting with the hydrostatic pressure accelerates the rate of inflow. The expression then becomes

$$\frac{\frac{P}{\rho}+H}{\frac{P}{\rho}+h_1}=e^{mt_2},$$

where t_2 is time of inflow.

Readjustments are made so that $h_1=0$, and H becomes H_1,

giving $$1+\frac{\rho H_1}{P}=e^{mt_2},$$

from which $$\rho=\frac{P}{H_1}\{e^{mt_2}-1\};$$

but $$m=\frac{1}{t_1}\log_e\frac{H}{h_1},$$

hence $$\rho=\frac{P}{H_1}\left\{e^{\frac{1}{t_1}\log_e\frac{H}{h_1}t_2}-1\right\}.$$

With the apparatus used a value of the density of water was obtained with an error of less than 1 per cent. This result justifies the use of the method, since no higher accuracy was

* Phil. "Trans.," 1894, A, p. 1.

attempted. The viscosities of the solutions used in the extreme cases are as 1 : 40.

Water is taken as a standard liquid of known viscosity.

Variation of Conductivity and Fluidity with Concentration at Constant Temperature.

The following table shows the results obtained for solutions of different concentrations all at the temperature of 16·77°C.

This temperature was approximately that of the room. To ensure its constancy the solution was contained in a cylindrical Dewar vessel, which, however, was not silvered. The temperature of the contents was brought to the above exact value by presenting to the walls of the vessel a warm or cool surface, and the liquid was stirred to secure uniformity of temperature. The vessel was then screened from further radiation.

TABLE I.

Temperature 16·77°C.

(1) Concentration. (n) Mol. per 100 mol. of water.	(2) Density. Grammes per cubic cm.	(3) Resistance. Ohms.	(4) Conductance. (C) $Ohms^{-1} \times 10^{-4}$.	(5) Viscosity. C.g.s. units.	(6) Fluidity. (F) C.g.s. units.	(7) $\frac{1}{n} \cdot \frac{C}{F} \times 10^{-}$
0	0·9988	...	...	0·01081	92·50	...
0·927	1·0434	2,032	4·921	0·0121	82·64	6·424
2·757	1·1293	915·4	10·93	0·0161	62·11	6·38
4·106	1·1793	782·5	12·78	0·0199	50·25	6·19
5·436	1·2313	766·8	13·04	0·0251	39·84	6·02
7·252	1·2933	845	11·83	0·0371	26·95	6·05
8·429	1·3308	968·5	10·32	0·0486	20·58	5 95
9·363	1·3569	1,074	9·31	0·0602	16·61	5·99
10·11	1·3788	1,179	8·479	0·0706	14·16	5 93
10·65	1·3960	1,278	7·823	0·0818	12·22	6 01
11·22	1·4063	1,435	6·969	0·0989	10 11	6 14
*16·66	1·5210	3,220	3·106	0·3320	3·012	6 18

* Melted crystals supercooled about 13°C.

The meaning of the last column is difficult to interpret, but is worth quoting. It will be seen that while the concentration alters from 4 to 16 molecules, while fluidity changes to 16 times its value and conductivity suffers a fourfold change, the quantity $\frac{1}{n} \cdot \frac{C}{F}$ only varies about 3 per cent.

A similar treatment of Lyle and Hosking's results for sodium

chloride and Hosking's results for lithium chloride suggests a similar conclusion. The data supplied, however, are inadequate, since the solutions are generally more dilute.

Curves connecting C and n, F and n, and $\frac{C}{F}$ and n are plotted in Fig. 3.

From the fluidity and conductivity curves no evidence is given of a vanishing of these quantities at higher concentrations.

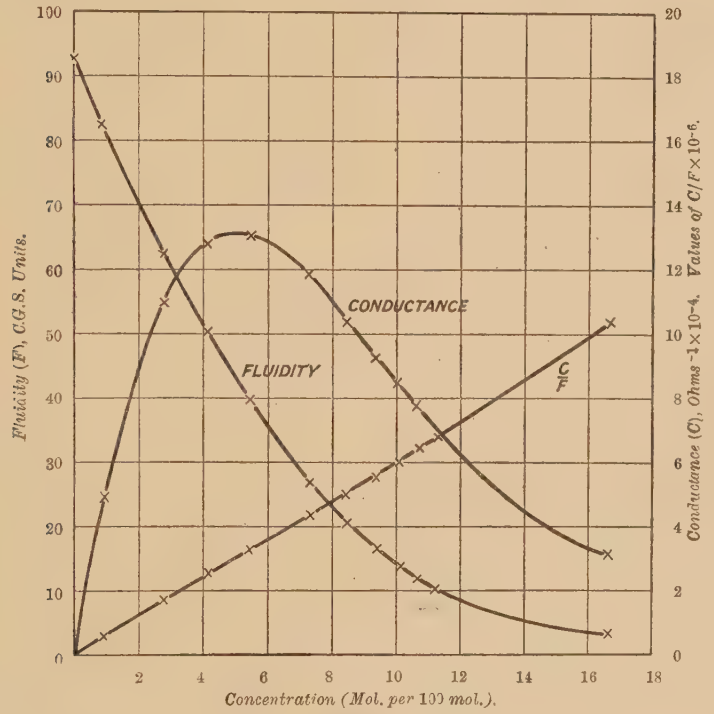


FIG. 3.

The above authors quoted such a vanishing of both quantities for sodium chloride at 10·74 normal, and for lithium chloride at 16-normal concentration.

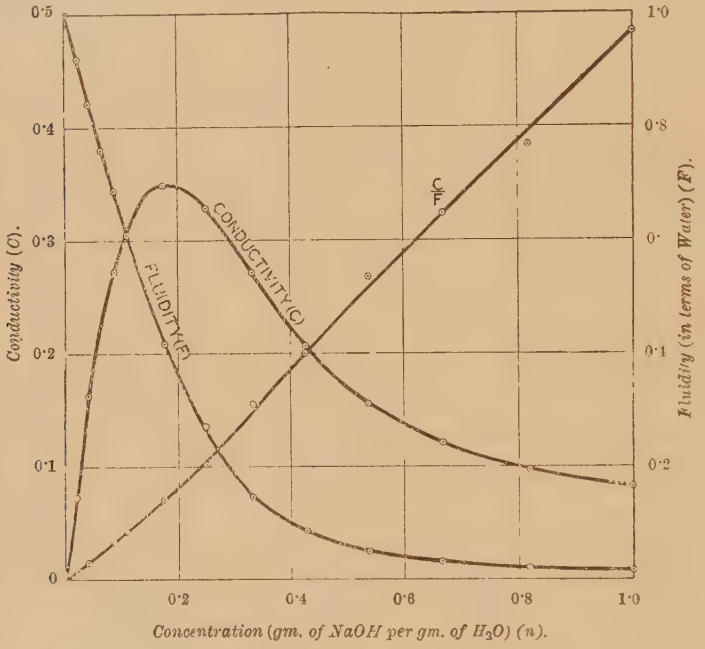
Both these results are obtained by extrapolation.

[Since this Paper was first submitted, the results obtained by Bousfield and Lowry* with strong solutions of sodium

* Bousfield and Lowry, "Phil. Trans.," 1905, Vol. 204, p. 253.

hydrate have been examined. The relations between conductivity, fluidity and concentration expressed as a ratio of weights of solid to solvent are shown in Fig. 3A.

The linear relation between $\frac{C}{F}$ and n for strong solutions is again shown.]



n	$\frac{C}{nF}$	n	$\frac{C}{nF}$
0·111	4·57	0·538	5·98
0·25	4·86	0·666	5·84
0·333	5·63	0·818	5·97
0·428	5·60	1·000	5·79

FIG. 3A.—PROPERTIES OF NaOH SOLUTIONS (BOUSFIELD AND LOWRY).

Variation of Fluidity and Conductivity with Temperature.

The same apparatus was used as before, except that the electrode P in the electrolytic cell was slightly moved.

The temperatures above that of the air were obtained by surrounding the Dewar cylinder by a second larger Dewar cylinder containing water.

This was heated by a current of steam.

For temperatures below that of the air, ice was added to the water, and below 0°C. a mixture of ice and salt was employed. Finally, for temperatures below −10°C. liquid air was used. After cooling to the desired temperature the excess of liquid air was boiled off by passing a current of air through it. During the observations the mean of the initial and final temperatures gave the temperature quoted.

In the viscosity-temperature experiment the solution of strength 7·252 molecules was taken. A preliminary experiment was made to determine the densities at the various temperatures employed. For this purpose an accurate mercury thermometer, reading to 50°C. by $\frac{1}{10}$ deg. divisions, was employed. The mercury was replaced by the solution and the volume of the bulb was accurately found by weighing the mercury.

The co-efficient of expansion for a range of nearly 100°C. is nearly constant, its mean value between 0°C. and 50°C. being 0·000455.

The following table gives the relation between fluidity and temperature :—

TABLE II.

Concentration 7·252 molecules per 100 molecules of water.

(1) Temperature. °C.	(2) Density.	(3) Viscosity.	(4) Fluidity. C.g.s. units.
85·22	1·259	0·01438	69·55
70·61	1·266	0·01678	59·59
56·79	1·275	0·01849	54·09
37·15	1·282	0·02666	37·51
23·70	1·290	0·03544	28·22
16·77	1·293	0·0418	23·92
−0·15	1·301	0·0726	13·77
−5·51	1·303	0·086	11·56
−8·85	1·305	0·1007	9·94
−15·19	1·308	0·1338	7·475
−19·07	1·309	0·1569	6·372
−30·45	1·314	0·2628	3·806
−49·16	1·323	0·5919	1·689

At the same time, readings of resistance were taken from which the conductivity for the same solution was obtained. These are included in Table III. (p. 120), together with results for the other solutions.

Columns (1) and (4) of Table II. and (1) and (2) of Table III. are plotted in Fig. 4, while columns (1) and (3) of Table III. are plotted in Fig. 5.

TABLE III.

(1) Temperature. °C.	(2) Specific conductivity of solution 7·252 mol.	(3) Values of $\frac{\text{conductivity}}{\text{concentration}}$ at the concentrations given.								
		0·927	2·13	5·436	7·252	8·22	9·363	10·11	10·65	11·22
39·72	2,280	1,034	922	455	314·0	250·1	198·0	166·7	150·3	132·4
31·74	2,000	909	808	398	276·0	217·0	170·2	144·0	143·4	112·2
27·67	1,858	853	742	371	256·0	200·0	157·6	130·5	119·2	103·4
23·69	...	...	...	...	...	...	...	...	108·0	92·0
19·74	1,598	739	628	320	220·0	170·0	143·6	110·0	...	85·1
15·78	1,457	677	558	295	201·0	153·3	121·0	100·0	88·3	77·7
11·70	1,340	617	...	268	185·0	142·1	106·6	89·5	...	69·0
9·00	...	...	492	...	...	...	...	...	...	...
5·79	...	522	...	...	...	...	...	...	...	57·1
4·09	...	...	...	...	...	111·5	86·0	...	...	...
3·81	...	505	...	222	...	...	...	...	...	...
−0·14	997	455	400	203	137·5	103·0	77·7	62·6	54·6	...
−4·37	880	...	...	180	121·4	89·6	68·3	54·6	...	...
−8·28	774	...	340	159·5	106·6	78·9	...	...	...	...
−10·24	...	...	316	...	...	...	554·0	...	...	...
−12·20	683	...	...	142	94·2	69·2	...	...	...	...
−16·14	600	...	...	124·2	82·7	59·4	...	...	...	...
−20·17	512	...	...	107·8	70·7	50·8	...	...	...	...
−24·11	435	...	...	97	60·0	43·0	...	...	...	...
−27·99	369	...	...	79·3	50·9	35·7	...	...	...	...
−31·88	307	...	...	67·1	42·5	...	...	...	...	...
−33·94	...	...	...	...	...	26·3	...	...	...	...
−35·84	246	...	...	...	34·0	...	...	...	...	...
−39·78	199	...	...	...	27·5	...	...	...	...	...
−43·65	149	...	...	...	20·5	...	...	...	...	...
−47·55	114	...	...	...	15·7	...	...	...	...	...
−51·5	81·9	...	...	...	11·0	...	...	...	...	...

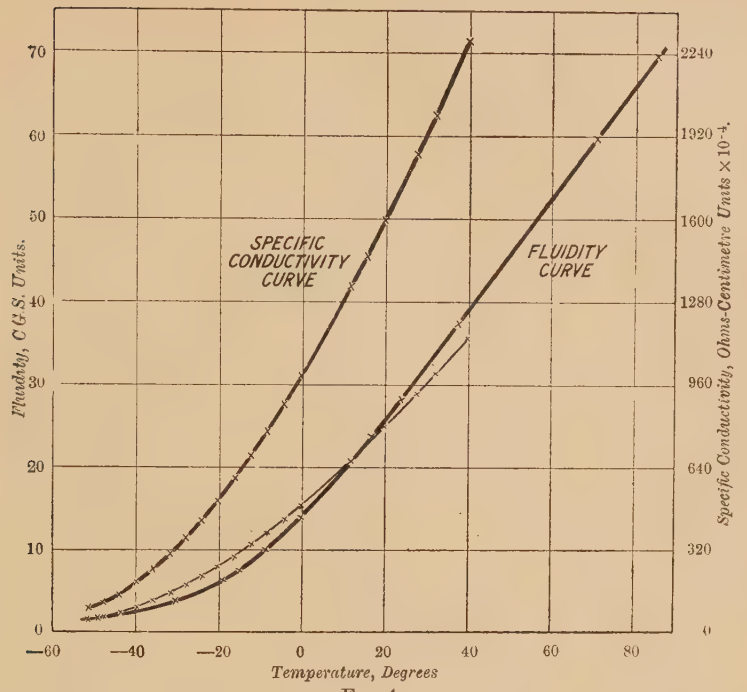

70
60
50
40
30
20
10
0
2240
1920
1600
1280
960
640
320
0
SPECIFIC CONDUCTIVITY CURVE
FLUIDITY CURVE
Fluidity, C.G.S. Units.
Specific Conductivity, Ohms-Centimetre Units × 10⁻⁴.
—60
—40
—20
0
20
40
60
80
Temperature, Degrees

FIG. 4.

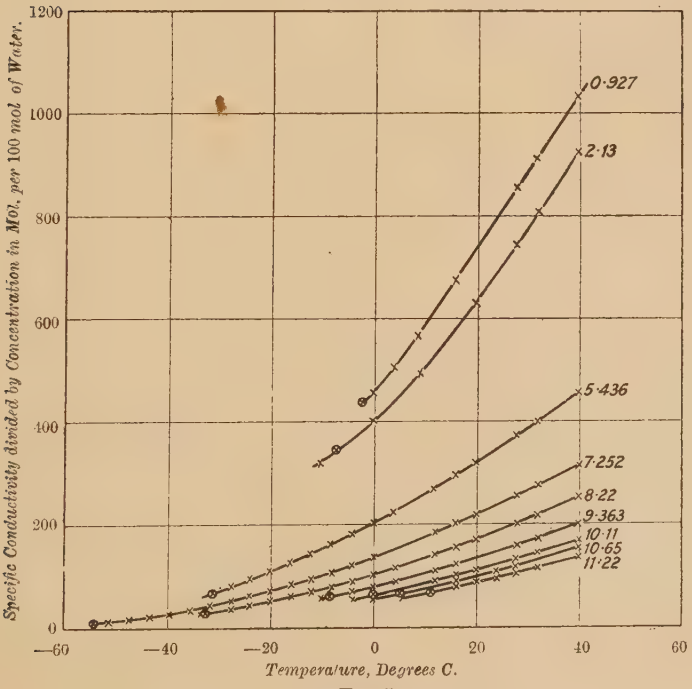

1200
1000
800
600
400
200
0
Specific Conductivity divided by Concentration in Mol. per 100 mol. of Water.
0·927
2·13
5·436
7·252
8·22
9·363
10·11
10·65
11·22
—60
—40
—20
0
20
40
60
Temperature, Degrees C.

FIG. 5.

Fig. 4 has a third curve, the ordinates for conductivity being halved, so that its variation with temperature can be more readily compared with that of fluidity.

The variation of conductivity with concentration and temperature is clearly shown in Fig. 5.

Very dilute solutions give a straight line relation, which is gradually modified to a curve of increasing curvature, as the concentration increases. The ratio $\frac{\text{conductivity}}{\text{concentration}}$ has here no definite meaning, since the denominator deals with a ratio of masses, but the latter has the advantage of not changing with temperature, and in this way the curves can be separated. They are very similar to the molecular conductivity curves.

It will be seen that there is no evidence of a disappearance of conductivity at a definite temperature, such as has been suggested by Lyle and Hosking for sodium chloride solutions and by Hosking for solutions of lithium chloride. The risk in extrapolation from above $-20°$C. to obtain such a vanishing point is clearly shown by the lower curves of Fig 5.

The above results also agree in this matter with those of Kunz,* who worked with sulphuric acid at low temperatures. As distinct from sulphuric acid, however, these properties of calcium chloride solutions exhibit no marked breaks in continuity with concentration at the temperatures above referred to. We thus have no evidence, within the limits of accuracy of the above observations, of the existence of many different hydrates.

The author wishes to thank Prof. Callendar and Dr. S. W. J. Smith for their valuable suggestions and helpful interest in the work, which was carried out at the Imperial College of Science.

ABSTRACT.

In adopting Callendar's association theory of strong solutions ("Proc." Roy. Soc., A., Vol. LXXX., p. 466) some difficulty is experienced in getting the strongest solutions of electrolytes to conform to the laws laid down. This is attributed to the inaccuracy of the ionisation data, which are derived from observations on electrical conductivity. It may be supposed that the viscosity of the solution will affect its conductivity, and the author carried out a series of experiments to determine if there were any definite relation between conductivity and fluidity in the case of calcium chloride solutions.

* Kunz, "Compt. Rend.," Vol. CXXXV., p. 788, 1902.

The feature of these determinations is the simultaneous observation of viscosity, electrolytic resistance and temperature.

Solutions were contained in an unsilvered Dewar cylinder, which permitted a slight adjustment of temperature if necessary.

A platinum thermometer records the temperature, and is caused to oscillate in the solution by a stirring mechanism.

In so doing it draws a platinum scoop through the liquid and thus acts as an efficient stirrer. The conductivity cell and the viscometer were bound to the thermometer by rubber bands. While the thermometer oscillates the readings of electrical resistance were measured by means of the rotating commutator and bridge. The viscometer was in the form of a capillary pipette, which was immersed in the solution a known depth. The levels of the inflowing liquid were indicated electrically, and hence the rates of inflow could be accurately estimated and compared with that of water. Viscosities correct to less than 1 per cent. were obtained.

Isothermal Observations.—Perfectly smooth curves for conductivity and fluidity were obtained, even when the supercooled melted crystals were included. No definite connection between conductivity, fluidity and concentration can be derived if the latter is expressed in terms of volume, but if concentration is expressed as a ratio of masses—molecules of solute to 100 molecules of solvent—the ratio conductivity C/fluidity F stands in linear relation to the concentration n when the latter exceeds one-fourth its maximum value. In spite of the enormous variations of the quantities in this ratio C/nF has values differing at most only 2 per cent. from the mean.

Examination of similar results by Bousfield and Lowry for sodium hydrate give striking agreement with this conclusion. However, owing to the dependence of C and F on linear dimensions, this relation is difficult to interpret.

Variations with Temperature.—One solution of nearly cryohydric strength was examined at temperatures from 40°C. to −50°C. The same tube and apparatus were employed, and for lower temperatures liquid air was used as a cooling agent. The failure of the fluidity-temperature and conductivity-temperature curves to exhibit the same variations was clearly shown.

Conductivities of various solutions were examined from 40°C. to their freezing points and the curves C/n and temperature plotted. The increasing curvature with concentration is shown and the error involved in applying the ratio, molecular conductivity to that at infinite dilution, obtained at one temperature, to indicate ionisation at another temperature, is quite apparent.

Moreover, within the limits of accuracy employed no indication is given of the existence of different hydrates.

The results obtained above suggest that no reliance can be placed on ionisation data derived from electrical conductivity observations.

DISCUSSION.

Prof. C. H. Lees remarked that the observations should be extended to see whether other strong solutions gave a simple relation between the fluidity and the resistance.

Mr. F. P. Worley thought the ratio of the conductivity to the concentration when the latter was expressed as so many gramme molecules of salt per 100 of water had a rather doubtful meaning.

Mr. F. E. Smith asked if Mr. Tucker had got any explanation of the fact that he found the cryohydric point $2\frac{1}{2}$ deg. higher than previously found.

The Author, in reply, stated that Morse and Fraser, working on the osmotic pressure of sugar solutions, found it bore a simple relation to the concentration per 100 molecules of water and not to the volume concentration. In reply to Mr. F. E. Smith, Roozeboom had used an alcohol thermometer in determining the cryohydric point. He might also have got it too low, due to supercooling.

XII. *Some Methods of Magnifying Feeble Signalling Currents.** *By* S. G. Brown.

Telegraphy over long submarine cables is continually on the increase, and I think it may be brought forward as a fairly accurate statement that the number of messages sent doubles itself every ten years. It is therefore important that, besides the increase in the number of the cables laid down each year, means should be devised to increase the carrying power.

The instruments which I have invented and am about to describe were designed primarily for cable work, but they are equally applicable to recording many other kinds of signalling impulses.

For good reasons, recording by photographic means is objected to by nearly every telegraphist. If the photographic method were permissible, great advances in speed would be available, but it is important that the record should be of a simple, cheap and immediate nature.

Lord Kelvin invented the siphon recorder in 1867—that is, about 45 years ago; he designed it so carefully that no improvement in its sensitiveness has been brought about until now.

Short Siphon Recorder.

In siphon recorders of the moving coil type what has to be done consists of—

1. Overcoming the inertia of the coil and siphon.
2. ,, ,, back E.M.F. of the coil.
3. ,, ,, control of the suspensions.
4. ,, ,, friction of air, suspensions and inking.

As the siphon has to return to zero in a certain time after the current in the coil ceases, it is necessary for the coil and siphon to have a definite frequency of oscillation depending on the speed of the signals. For submarine telegraphy this frequency lies between about 3 and 10 per second, and is adjusted by varying the control on the coil. As the control necessary to give a certain natural period to the moving system is proportional to its moment of inertia, it follows that by reducing this inertia we reduce the forces required both to accelerate the coil and to overcome the control.

* Discourse delivered at the Eighth Annual Exhibition of Apparatus, held by the Society on December 17, 1912.

The ordinary siphon recorder employed is a siphon tube of about $2\frac{1}{2}$ in. long and from 8 to 12 mils in diameter. The moving coil consists of 500 turns of 2-mil wire at a mean radius of $\frac{3}{8}$ in. The coil and siphon are mounted on separate axes and are connected by silk fibres so that the angular movement of the siphon is about two to three times that of the coil. By reducing the length of the siphon to $\frac{1}{2}$ in. and substituting a narrower coil it is possible to greatly increase the sensitiveness of the recorder.

In order to make the inertia effects of the moving system a minimum, it is advisable to make them equal for the coil and the siphon. Even a narrow coil of 300 turns has about 100 times more inertia than the siphon, so that it is necessary to

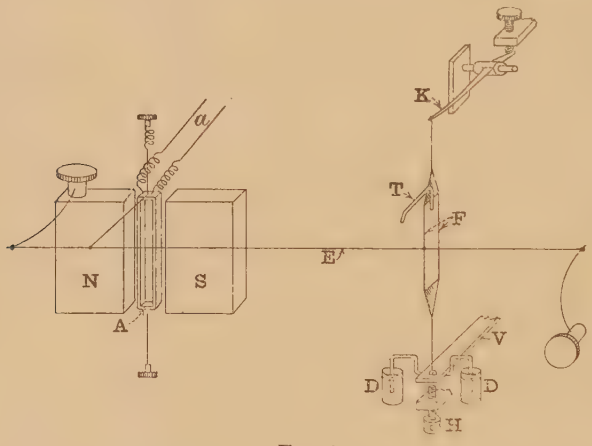


Fig. 1.

move the siphon through $\sqrt{100}$ times the angle moved by the coil.

By reducing the number of turns on the coil and increasing the field it is possible to reduce the natural period for a given sensitiveness and back E.M.F., but as the mass of the mountings and insulation of the coil only decrease slightly as the turns are reduced the gain is not very marked. In practice it is inadvisable to reduce the turns on the coil below 50 or 100 turns, as with lower values the power required to overcome the friction of the air and inking becomes too limited. This precludes the possibility of attaching the siphon directly to a coil of a few turns, and means of magnifying the motion of the coil and transmitting it to the siphon have to be used. In this instru-

ment (Fig. 1) it is accomplished by means of a fine fibre, E, which is kept in tension by flat springs at each end. The fibre is attached to an arm carried by the moving coil A, and to a vertical fibre, F, on the siphon suspension.

The siphon is carried on an aluminium carrier to which a single central fibre is attached at the top and two parallel fibres, FF, 0·2 in. apart below. One leg of the siphon (Fig. 2) lies on the axis of the suspension and dips into a small opening in a pipe extending from the ink-pot. This arm goes in between the two vertical fibres, and as the opening in which the siphon dips is only a small one, the ink level remains practically constant whether the reservoir is full or not. The siphon turns round on the axis in which the leg lies, and this makes the drag

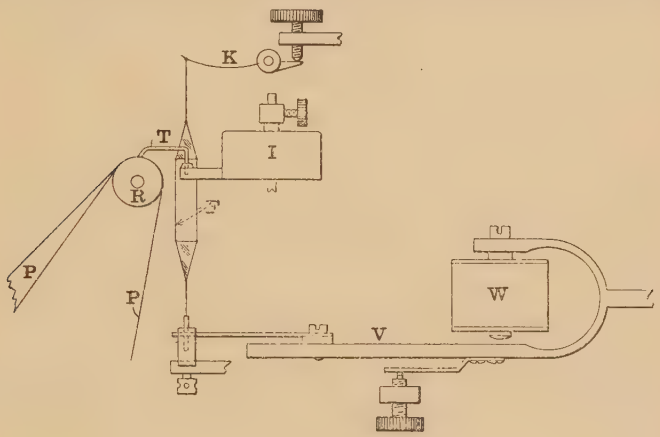


Fig. 2.

between the moving siphon and the ink very much less than if the siphon cut across the surface of the ink.

In order to produce an ink line on the paper without introducing friction the siphon must not touch the paper even momentarily, and arrangements have been made to jerk the ink in fine drops on to the paper. To accomplish this the whole of the siphon suspension is vibrated rapidly up and down between the springs V and K by means of the spring V, which is attached to the vibrator. As the spring V is very weak in comparison with the reed, the vibrations of the latter are not affected by the movements of the spring. To impart a jerk to the siphon a stop, H, is fixed directly under the axis of suspensions and two little dash-pots, DD, on either side prevent the spring bouncing on the stop.

The working end of the siphon is ground flat, and an aniline dye with a small proportion of methylated spirit or ordinary red ink is used for recording on the paper. In this way a fine line of very closely spaced dots can be obtained without introducing any appreciable drag on the siphon.

For signalling purposes, the distortion due to the radius of the siphon being only $\frac{1}{2}$ in. is not at all troublesome as the velocity of the paper moving round the wheel R masks this.

When the instrument is adjusted to have a natural frequency of 10·5 per second, with a 300-ohm 300-turn coil, a current of 50 microamperes gives a full-sized signal corresponding to a deflection of 0·1 in. on the paper. Under these conditions the back E.M.F. of the coil is only about one-quarter to one-fifth of that of the ordinary recorder coil.

Trials with this instrument have shown an increase of speed of 30 per cent. on the largest Atlantic cables.

Thermo-electric Magnifying Relay.

In this instrument (Fig. 3) the power in the relay circuit is generated by means of five thermo-junctions at different temperatures. The heat is supplied by two little flames, CC, and a very light thermopile, B, is suspended so as to swing in and out of the flames and is coupled to a moving coil through which the received currents pass.

The thermopiles consist of alternate junctions of platinum and platinum + 20 per cent. iridium, wires being used of 1 mil diameter. The joints are made by twisting the ends of the two wires together and holding the junctions in a Bunsen flame for a short time. In this way a perfectly good and permanent joint is ensured. The wires are melted on to a fine glass tube about 10 mils in diameter, and one connection is brought down inside the tube to the first junction and the other connection comes along the outside of the tube.

For moving the thermopile in the flames similar arrangements to those just described for the siphon recorder are employed. Under the saddle which carries the thermopile the two silk fibres are stretched, and on to one of these the cross fibre which transmits the movements of the coil to the thermopile is attached. The top and bottom suspensions are of fine phosphor bronze wire and serve as leading-in wires to the thermopile.

To supply the heat two little flames are fed by two or three strands of cotton wick with alcohol or methylated spirit. If the

wick just protrudes above the opening a small steady flame is produced and the lamp is provided with adjustments to vary the distance between the flames and the position of both flames relative to the thermopiles.

Instead of burning directly on the lamp wicks, a simple vapour burner can be fitted which will give good results even with very impure spirit. This consists of a brass cap which is kept hot by a copper wire attached to it at one end, and is heated at the other end by the flame. By altering the amount of wire in the flame the size can be varied.

An alternative arrangement which gives greater sensitiveness and enables heavier thermopiles to be used is to fix the thermopiles and vary the flames by means of a valve or shutter actuated by the coil movements.

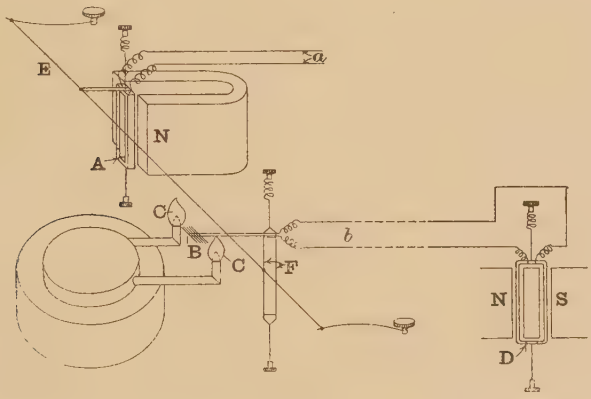


FIG. 3.

As the thermopile current depends on the difference of temperature between the junctions a certain time is required to heat the wires. It is found that for cable work, where the frequency seldom exceeds 10 per second, the lag is inappreciable, but for considerably faster movements it becomes important.

In duplex working when the sending current has to be balanced so as not to affect the receiver, quick " jarry " movements are very difficult to eliminate, but the lag in the thermo instrument reduces these movements very considerably and is a valuable property.

When the thermopile is in its central position and no current

is flowing both junctions are at a dull red heat, and when fully deflected one junction becomes bright red and the opposite one is black or very faintly red. In intermediate positions the current generated by the thermopile is nearly proportional to the deflection.

The curve (Fig. 4) was taken from a thermopile with seven junctions on each side. When the thermopile was deflected 0·075″ the current it sent through a resistance of 42 ohms (equal to its own resistance) was 0·81 milliampere. With the natural period of the coil equal to 8·7 per second and a 480-ohm 480-turn coil, a current of 0·03 milliampere through the coil gave a current of 0·81 milliampere from the pile through an external

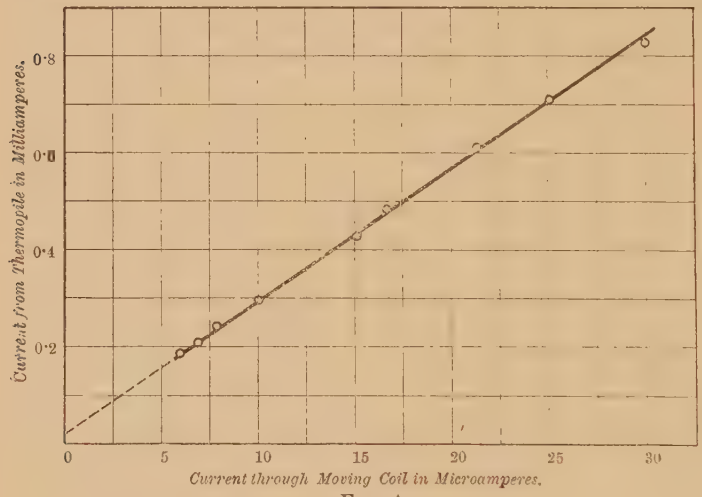


Fig. 4.

resistance of 42 ohms. For slowly changing currents this corresponds to a magnification of power of about 27 times, and of course this can be greatly increased by reducing the period of the coil. For quickly changing movements the power magnification is not so great, owing to the back E.M.F. of the coil.

Trials of this instrument on an Atlantic cable have shown an increase in speed of about 40 per cent.

Mechanical Relay.

The instrument just described is a magnifying relay—that is to say, it multiplies the impulses received in exact proportion

to their strength. This form of relay is quite distinct from an ordinary make-and-break relay, which delivers a constant current for any impulse over a certain strength. For very many purposes it is essential that received impulses should be magnified without altering their shape, and this can only be done by an instrument with a constant magnifying power.

That this is the case in the thermo relay is shown by the diagram (Fig. 4) where the current supplied to the coil and the current delivered by the thermo-junctions are plotted. Within the range of the instrument the points lie on a straight line and represent, in this case, a constant magnification in current of about 27 times.

This property I will now illustrate in an entirely mechanical relay in which movements operated by very small forces are

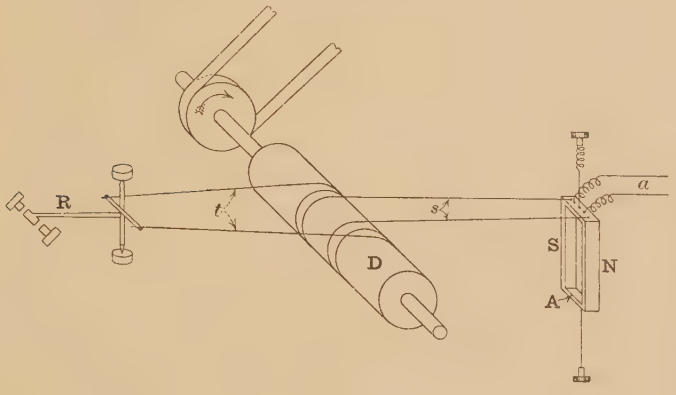


FIG. 5.

largely increased in strength without affecting their motion. The relay consists in principle of a rotating spindle around which are wound one or more turns of a flexible cord. The spindle is revolving in such a direction as to pull away from the magnified forces and towards the small forces that control the movement. Suppose a heavy weight has to be raised by a force of one-tenth of the amount, it will obviously be necessary to supply 90 per cent. additional energy, and this is supplied by the motor driving the spindle. The magnification of force and energy depends on the number of turns which the cord makes round the spindle and follows a compound interest law.

In the model shown it will be seen that a large magnification of power can be easily obtained by very simple means. Thus

I can move this 14 lb. weight rapidly up and down by pulling upon this silk fibre.

Fig. 5 shows an application of the principle to cable work, in which the small forces operating the coil A are intensified sufficiently to work the coarse relay arm R. The spindle rotates away from the relay arm R and towards the coil, and produces a much greater tension in the fibres *t* than in *s*. When the coil swings on its axis the tension is increased in one of the fibres and diminished in the other and a similar change in a magnified degree takes place in the fibres *t*.

By using means of this sort it is possible to work an ordinary siphon direct writer which normally requires some 3 milliamperes by a current of 10 microamperes

XIII. *The Resistance of Electrolytes.* *By* S. W. J. Smith, *M.A.*, *D.Sc.*, *and* H. Moss, *B.Sc.*, *Imperial College of Science.*

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I. *Introductory.*

In 1911 some interesting experiments were exhibited before this Society from which the conclusion was drawn that the resistance of an electrolyte subjected to alternating currents varies with the frequency of these currents to an easily perceptible degree. A bridge method was used, a variable self-induction being inserted in the arm containing the electrolyte and a vibration galvanometer being employed as detector.*

It was assumed that the E.M.F. of polarisation could be written in the form

$$e=\frac{1}{C}\int idt,$$

in which i is the current and C the so-called capacity of the electrodes.

Now this expression takes no account of the fact that few, if any, electrolytic condensers retain their charges unimpaired when the polarising current is removed. There is in general a leakage which increases with the degree of polarisation of the electrodes. It is easy to show that the effect of such a leakage is to alter the phase of e with respect to i, so that they are not in quadrature as the above expression supposes.

In the simplest case, when $i=i_0 \sin pt$, the value of e is not

$$\frac{-i_0}{Cp}\cos pt,$$

which it would be if there were no leakage, but

$$\frac{-i_0}{C'p}\cos(pt+\varphi),$$

where C′ is obviously greater than C because the maximum value of e is less than it would be if leakage were absent.

The effect of this change in the expression for e is that (when the current $i=i_0 \sin pt$ is passing) the E.M.F. between the ends

* H. F. Haworth, "Phys. Soc. Bulletin," March, 1911.

of the arm (of total resistance R) containing the electrolyte is not

$$E = i_0\left\{R \sin pt - \left(\frac{1}{Cp} - Lp\right)\cos pt\right\}$$

but

$$= i_0\left\{\left(R + \frac{\sin\varphi}{C'p}\right)\sin pt - \left(\frac{\cos\varphi}{C'p} - Lp\right)\cos pt\right\}.$$

The condition that E and i should be in the same phase—and therefore that their ratio should be the same at any instant—is thus not

$$\frac{1}{Cp} - Lp = 0$$

but

$$\frac{\cos\varphi}{C'p} - Lp = 0.$$

Hence, when the bridge is balanced, the apparent resistance of the electrolyte arm is not R but

$$R\left(1 + \frac{\sin\varphi}{C'pR}\right).$$

It will therefore appear to be a function of p if the quantity $\sin\varphi/C'pR$ is not negligible in comparison with unity.*

The interactions between polarised electrodes and the surrounding solutions are frequently complex and the phenomenon just indicated is only one out of several possible disturbing influences. It is difficult in any particular case to decide that all of them have been allowed for.

The cause of the leakage assumed above varies from one case to another; but it can be followed very clearly in one particular case which we shall attempt to summarise.

Consider a cell in which the electrolyte contains a salt of the metal of which the electrodes are composed. Let the concentration of the salt be c and let a small E.M.F. be applied to the cell. The current which ensues will be accompanied by changes of concentration in the border layers of the electrolyte. Let these changes be $+dc$ and $-dc$ respectively at the anode and cathode surfaces. Then, assuming the logarithmic formula due to Nernst, the expression $e \propto dc/c$ will represent, sufficiently closely, how the back E.M.F. of polarisation depends upon dc. Using alternating currents and sup-

* For estimates of the magnitudes which $\sin\varphi/C'$ can attain, *see* M. Wien, "Ann. der Physik," LVIII., p. 37, 1896; *also* F. Kohlrausch, *ibid.*, p. 514.

posing diffusion to be negligible, the concentration changes, and the back E.M.F., would acquire maximum values at the end of each half-period of the current, as the expression $e \propto \int i dt$ requires. But it is obvious that diffusion cannot be ignored. Its effect will be to make the concentration changes reach maximum values before the end of each half-period—namely, when the rates of gain and loss of the cathode and anode layers respectively, owing to diffusion, equal the rates of loss and gain due to the current. If we confine the concentration changes to one electrode by making the other electrode very much larger we get for the stationary state, as Warburg has shown,*

$$dc \propto \frac{-i_0}{\sqrt{p}} \cos\left(pt+\frac{\pi}{4}\right),$$

where, as before, dc is the change of concentration in the immediate neighbourhood of the electrode. If we suppose that, by the addition of a neutral salt containing the same anion, any P.D. within the electrolyte is obliterated, we get that

$$e \propto dc/c \propto \frac{-i_0}{\frac{c}{\sqrt{p}}p} \cos\left(pt+\frac{\pi}{4}\right).$$

Comparing this with the expression $\frac{-i_0}{C'p} \cos(pt+\varphi)$ for e given on p. 133, we see that we ought to have $\varphi=\pi/4$, that the apparent capacity $K=1/Lp^2=C'/\cos\varphi$ should be proportional to $c/\sqrt{p}$, and that the variable part of the apparent resistance, *i.e.*, $\sin\varphi/C'p=Lp\tan\varphi$, should be Lp.

These conclusions can be verified experimentally, as has been shown by Neumann.†

Next, suppose that c is reduced until it becomes almost negligible. The changes of concentration dc accompanying a given small polarisation e will now almost vanish. Diffusion effects will be reduced to a minimum, since the concentration gradients will have almost disappeared, and the apparent capacity due to concentration changes will be negligible. The polarisation (variations in the charge) of the condenser-like double layer at the electrode surface will, however, take place as before. The capacity of this layer (neglected above) is inde-

* E. Warburg, "Ann. der Physik," LXVII., p. 493, 1899.
† E. Neumann, "Ann. der Physik," LXVII., p. 499, 1899.

pendent of c, and occasions a back E.M.F. of the simple form first mentioned. It is, therefore, to be expected that, as c gets smaller, φ will approach zero and the capacity a relatively small constant value independent of p. The effects produced by gradually reducing c have been studied experimentally by Krüger,* and shown to be explicable in the way here indicated.

We have thought it necessary, however imperfectly, to draw attention to this work, because it seems to have attracted much less notice than it deserves. It shows the need for care in the interpretation of any new experiments upon the resistance of electrolytes.

II. *Experimental.*

It occurred to us that there was a simple way in which we could use modern instruments to find, without risk of serious error, whether the resistance of an electrolyte depends appreciably upon the frequency of the currents to which it is subjected. It depends upon direct readings of current and voltage and, consequently, is not susceptible of the highest accuracy. We thought, however, that we could expect to attain an accuracy of 1 part in 1,000 in comparing resistances, at any frequencies within the range at our disposal, by its use.

The principle of the method will be obvious from the figure (representing a particular case) given on p. 138, and is, in fact, well known. What novelty there is lies in the manner in which the principle is applied.

Currents measured by an ammeter A are passed through the electrolyte by means of main electrodes p and q. The resulting P.D.s between the ends of a tube containing the electrolyte are measured by means of the voltmeter V and the auxiliary electrodes p' and q'. The ratio of the two readings gives the resistance.

The procedure when using direct currents requires little comment. We have merely to ensure that the voltmeter gives, to a sufficient degree of accuracy, the P.D. between the ends of the tube when a known current, measured by A, is passed through it.

For this purpose it is necessary to use a voltmeter of which the resistance is extremely great compared with that of the electrolyte and of which the capacity can be neglected in com-

* F. Krüger, "Zeits. f. Physik. Chemie," XLV., p. 1, 1903. For another aspect of polarisation capacity, *see* E. Rothé, "Ann. de Chim. et de Phys.," [8], Vol. I., p. 215, 1904. *Cf.* also Krüger, "Physik Zeitschr.," XI., p. 719, 1910.

parison with that of the auxiliary electrodes p' and q'. If an electrostatic voltmeter is used the resistance is, of course, practically infinite and the capacity, even of a very sensitive instrument, is very small compared with that of electrodes of moderate size. For example, the minimum capacity of electrodes of such metals as Hg, Cu and Pt is of the order 10 mfd. per square centimetre, while that of the electrometer may be of the order 10^{-4} mfd. or less.

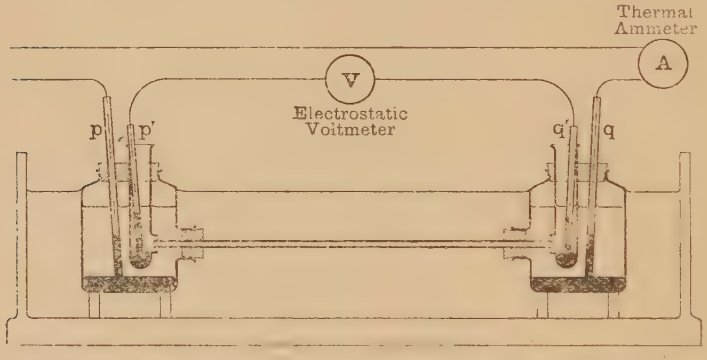
It is further necessary that the contact P.D.s between p' and q' respectively and the electrolyte should not be subject to uncertain fluctuations. For this reason it is best to use mercury electrodes whenever possible. The two contact P.D.s will then usually be not only constant but also of practically identical magnitude. Any appreciable difference between them would, of course, affect the measurements. It can be allowed for, if present, by taking the mean of the voltages obtained when the current is passed through the electrolyte first in one direction and then in the opposite. In making this latter test it may be necessary to allow for certain peculiarities of the electrometer which will be described later.

Suppose now that alternating currents are being used. It is again necessary that the capacities of the auxiliary electrodes should be very large compared with that of the electrometer ; but, in addition, the quantity $2\pi^2n^2C^2R^2$, where n is the frequency of alternation, C the capacity of the electrometer and R the resistance of the electrolyte, must now be small in comparison with unity. This expression shows that the capacity of the electrometer may require to be taken into account if the frequency and the resistance are high. For instance, if the frequency is of the order 2,500 $\sim$ per second and the resistance of the order 10,000 ohms, the capacity of the electrometer must be allowed for if it exceeds 2×10^{-4} mfd. when an accuracy of the order 0·05 per cent. in the measurement of R is desired.

In our case it was advisable to work with fairly high resistances because the ammeters which we had gave a much wider range of sensitivity than the voltmeter. Other things permitting, it is an advantage to use high resistances. End corrections may then be made inappreciable and, working with small currents, it is possible to reduce the disturbance due to the heating effects of the latter to a very small quantity. Further, it is relatively easy when working with continuous currents to keep these steady for a considerable time, when they are small, without using main electrodes of large area.

The current-measuring instruments at our disposal were a Duddell thermo-ammeter suitable for currents of the order 10 milliamperes and a Duddell thermo-galvanometer of which the sensitiveness could be varied within wide limits. The available voltmeter was an Ayrton-Mather electrometer (constructed by Paul) of which the deflection could not conveniently be read, with an accuracy of 0·1 per cent., unless an E.M.F. of the order 10 volts was applied between its terminals. Its capacity was, however, sufficiently small to be negligible when using the instrument in parallel with a resistance of 10,000 ohms at the highest frequency of alternation available.

We had three convenient means of obtaining alternating currents, namely, a rotating commutator with which any



frequency between 0 and 100 ∽ per second could be obtained, a Crypto alternate current generator giving conveniently 100 ∽ per second, and a Siemens high-frequency alternator giving frequencies of 500 to 2,300 ∽ per second. The latter were measured by means of a Frahm vibrating-reed frequency indicator.

In the experiments to be described the electrolyte was contained in a tube, of about 1 mm. internal diameter and about 30 cm. long, connecting two bottles in which were the main and auxiliary electrodes as shown in the figure above.

The area of each main electrode was about 30 sq. cm. Each auxiliary electrode had an area of about 2·5 sq. cm. and was contained in a test tube perforated at the side, supported as shown in the figure. This apparatus was immersed in an oil bath by which sudden changes of temperature were prevented.

The circuit was made as simple as possible, to avoid

unnecessary self-induction and capacity, and contained only the electrolytic resistance, the measuring instruments and the source of E.M.F. Any desired change in the intensity of the alternating current was made by altering the magnitude of the exciting current in the generator.

The apparatus was arranged so that observations with continuous currents and with alternating currents of different frequencies could be taken one after the other in rapid succession. Readings were taken alternately with continuous and alternating currents for a period of half an hour so that a comparison between the two should not be affected by small changes of the galvanometer zero (which could not be entirely eliminated) or by a change in temperature. The continuous current was reversed between successive determinations. The current was adjusted approximately to the same value for each determination of the same series and several series with different currents were obtained. One series of readings is shown in detail in Table I. The temperature of the oil bath remained at 18·1°C. throughout the experiment. The electrolyte used was a normal solution of potassium chloride.

TABLE I.

Time.	Frequency, ~ per sec.	P.D., volts.	Current. amperes.	Resistance, ohms.		
				0	100 ~	500 ~
1.50½ p.m.	500	10·235	0·001127	...	...	9,081
1.51 ,,	0+	10·115	0·001116	9,064		
1.51½ ,,	100	10·07	0·0011085	...	9,084	
1.52½ ,,	0—	10·11	0·001115	9,067		
1.53½ ,,	500	9·805	0·001079	...	...	9,087
1.55 ,,	0+	9·95	0·001097	9,070		
1.56 ,,	100	10·015	0·001102	...	9,088	
1.56½ ,,	0—	10·12	0·0011155	9,072		
1.57½ ,,	500	9·955	0·0010955	...	...	9,087
2.0 ,,	0+	9·87	0·001088	9,072		
2.1 ,,	100	10·015	0·001102	...	9·088	
2.1½ ,,	0—	10·125	0·0011165	9,069		
2.3 ,,	500	9·805	0·001079	...	...	9,087
2.4 ,,	0+	10·02	0·001105	9,068		
2.4½ ,,	100	10·07	0·001109	...	9,080	
2.5 ,,	0—	10·105	0·001115	9,063		
			Means ...	9,068	9,085	9,085

It will be seen that the greatest difference between individual readings at each frequency does not exceed 0·1 per cent. and that the mean readings for each frequency are identical within

0·05 per cent. at least. The conditions of working were such that we could expect to be able to detect differences of this order. We aimed at getting identical readings of the ammeter in successive experiments at different frequencies and comparing the corresponding readings of the voltmeter. Since we could compare these with certainty within 1 part in 1,000 and since the deflection varies approximately as the square of the voltage we could expect to detect any variation of the voltage exceeding 1 part in 2,000. Thus we could usually compare the apparent resistances to within 0·05 per cent. without difficulty, and still more nearly under the most favourable conditions (absence of zero variations).

From the numbers in Table I. there can be no doubt that the resistances as measured by this method are identical at frequencies of 100 $\sim$ and 500 $\sim$ per second respectively.

We obtained a precisely similar result on comparing the resistances at frequencies of 500 $\sim$ and 2,300 $\sim$ per second. This is shown in Table II., in which two sets of observations similar to those of Table I. are summarised, the comparative resistances being given to the nearest 5 in 10,000 in each case.

Table II.

Approx. P.D. volts.	Approx. current, amperes.	Resistance.		
		0	500 $\sim$	2,300 $\sim$
10	0·001	9,425	9,440	9,440
13	0·0014	9,395	9,410	9,410

There was, therefore, no ambiguity about the results of the experiments with alternating currents. But when these are compared with the results for steady currents it will be seen that the latter are uniformly about 1 part in 600 lower. So that even if we assume that the experimental error may amount to 0·1 per cent. there is still a small difference of at least 0·05 per cent. which remains unaccounted for.

The easiest way of testing whether this difference arose out of the method of calibrating the instruments was to replace the electrolyte by a metallic resistance and to observe whether the same discrepancy recurred. For this purpose a resistance of about 700 ohms was constructed by depositing a thin film of gold upon a strip of glass.

It was now necessary to use currents of the order 0·014

ampere to get P.D.s of the order 10 volts. The results of two series of observations are shown below :—

TABLE III.

	Frequency.			
	0	100 ∿	500 ∿	2,300 ∿
Resistance I. ...	734·9	...	736·5	736·4
Resistance II....	733·4	734·9	735·0	...

Similar results were obtained with various other resistances. Differences of exactly the same order as those obtained with the electrolyte were always observed. (We found, using different steady voltages, that the resistance of the gold strip was always the same at the same temperature within the limits of experimental error.)

From these results it is obvious that the small differences exhibited in Tables I. and II. do not represent any peculiarity of the electrolyte and that the resistance of the latter was the same within 1 part in 2,000 whether it was carrying steady currents or alternating currents of any frequency up to 2,300 ∿ per second.

III. *Supplementary.*

It is perhaps desirable, in view of the behaviour of the instruments, that the method of calibration should be indicated.

The ammeter gave the same deflection when a known current was passed through it in either direction and the readings showed that the couple producing deflection of the suspended system was very nearly proportional to the square of the current. We therefore assumed that when an alternating current i, of period τ, produced the same steady deflection as a continuous current i_0, the relation given by the equation

$$i_0^2\tau = \int_0^\tau i^2 dt$$ was true.

The voltmeter, however, gave appreciably different readings when the same E.M.F. was applied to it in different directions. In our calibration, upon which the results given in the Tables depend, we assumed that this was due to a contact P.D. within the instrument which remained constant during the measurements. Calling this P.D. e we applied a known steady voltage v_0, first in one direction and then in the other, and assumed that the two deflections θ_1 and θ_2 could be written

$$\theta_1 = k_{\theta_1}(v_0+e)^2 \text{ and } \theta_2 = k_{\theta_2}(v_0-e)^2$$

respectively.

Since θ_1 and θ_2 were nearly equal and since we had reason to believe that $dk_\theta/d\theta$ was small, we took $k_{\theta_1}=k_{\theta_2}=k$. Whence we got $\frac{1}{2}(\theta_1+\theta_2)=k(v_0^2+e^2)$. We then assumed that if we applied an alternating voltage v, of period τ, satisfying the condition $\int_0^\tau v^2dt=v_0^2\tau$, we should get a steady deflection $\theta=\frac{1}{2}(\theta_1+\theta_2)$, since the mean couple tending to produce deflection in this case would be proportional to

$$\frac{1}{\tau}\int_0^\tau (v+e)^2dt, \text{ } i.e., \text{ to } v_0^2+e^2.$$

We have found, however, that a small error, which would produce differences of the same sign as those shown in Tables I. and II., can be introduced in this way. Because, even if it be correct to explain the difference between θ_1 and θ_2 by means of a contact E.M.F., it is possible to show that it is incorrect to assume that this E.M.F. is constant. In the instrument we used it decreased perceptibly when v was increased. For example, applying first a low steady voltage v_0, we inserted between this and the electrometer a small constant E.M.F. of such sign and magnitude that the voltmeter deflection remained unchanged when v_0 was reversed. The apparent contact E.M.F. was then balanced. Increasing the applied voltage until the deflection was approximately doubled it was easy to see that the contact E.M.F. was now over-compensated by the inserted voltage. An approximate estimate showed that, in one case, the apparent value of e decreased from about 0·07 volt when the applied voltage was 7 to about 0·06 when the applied voltage was 10.

Hence e is a function of the applied voltage. Its rate of variation may depend upon the sign of the latter; but unfortunately its relative magnitude, compared with the voltages required to produce accurately measurable deflections, was so small that it was impossible with our instrument to decide this question. For obvious reasons we are unable to say whether these peculiarities are shared by other instruments.

ABSTRACT.

Some experiments upon this question were exhibited before the Society by Dr. Haworth in 1911. In these a modification of Wien's method was used—the optical telephone being replaced by a vibration galvanometer—and the conclusion was drawn from them that the resistance of an electrolyte varies to an easily perceptible degree with the frequency of the alternating currents to which it is subjected.

The authors point out that the terminal difference of potential and the current in a branch of a network containing capacity and self-induction may be in the same phase although their ratio does not give the resistance of that branch. This will happen (whether the branch contains an electrolyte or not) if there is leakage through the condensers, causing the P.D.s between their plates to be out of quadrature with the current. The apparent resistance of the branch will then be a function of the frequency of the alternating currents circulating in the bridge, and Wien's method (as Wien himself points out) will give this apparent resistance only.

It is, therefore, unsound to use the method to test whether the resistivity of an electrolyte depends upon the frequency of the currents to which it is subjected, unless it is shown that the effects of leakage through the electrolytic condensers can be neglected or allowed for.

A particular case was indicated by the authors, in which the leakage could be made large or small at will. The results for this case had been interpreted by Krüger, without assuming any variation of resistance with frequency, in a manner which seemed satisfactory to them.

In order, however, to remove or justify any doubt upon the question they have performed test experiments by a simple and direct method which was described. It depends upon simultaneous measurement of the voltage between the ends of a tube containing the electrolyte and of the current passing through it. The former was measured by means of an Ayrton-Mather electrostatic voltmeter connected to auxiliary electrodes and the latter by means of a Duddell thermo-galvanometer.

In the cases examined it was found that the resistivity of the electrolyte was constant within 0·05 per cent., whether steady currents or currents of any frequency up to 2,300 alternations per second were used.

Until the instruments were calibrated by means of a metallic resistance there appeared to be a small difference of about 1 part in 600 between the resistance as measured by continuous currents and the values obtained with alternating currents.

Some supplementary experiments were made with the object of elucidating the peculiar behaviour of the instruments which this calibration disclosed. On account of the smallness of the effect its cause could not be completely ascertained; but the fact that the apparent contact P.D. within the voltmeter was a function of the applied voltage, decreasing as the latter was raised, would cause an effect of the same sign as that observed. Unallowed-for leakage, greater with steady than with alternating currents, might also provide a partial explanation of the results.

DISCUSSION.

Mr. A. Campbell expressed himself interested, and thought that the very small discrepancy described might be due to the method employed, which involved taking direct readings of both volts and amperes. He had not noticed the effect himself. It might be that it only occurred in particular instruments. He thought the effect was more likely to lie in the thermo-ammeter than in the electrostatic voltmeter.

Mr. F. E. Smith asked how long the current was left on before readings were taken, and was it possible that with direct currents the effect might be due to differences of concentration round the electrodes. He agreed, however, that the fact that gold showed the same phenomena seemed to negative this possibility.

Mr. R. S. Whipple suggested a radio-active leak in the voltmeter.

Prof. G. W. O. Howe thought the discrepancy was too regular to be due to any radio-active leaks.

Dr. H. F. Haworth remarked that Dr. Smith in his equations had assumed R to be independent of the frequency. He suggested that the resistance decreased as the frequency was raised because the complex molecules in the solution were shaken asunder, and also suggested that using platinised electrodes instead of ordinary platinum might have the effect of disintegrating these complex molecules in the solution, thereby giving a constant value of the resistance. The mercury electrodes employed by Dr. Smith might also have just the same effect. Heating the solution also decreases the resistance by breaking up the complex molecules.

Mr. G. D. West asked if the authors had used lower frequencies than 100.

Mr. G. L. Addenbrooke remarked that it had been known to him for a long time that contact P.D. was by no means constant. It could be altered by fingering the apparatus. He thought the deflections with alternating currents might be influenced by vibration of the electrometer needle.

Mr. F. P. Worley remarked that physical chemists were entirely dependent on physicists for the method they employed in measuring electrolytic resistances, and any suggestions for improving methods followed by chemists would be very acceptable to them.

Prof. C. H. Lees remarked that Dr. Smith had given a very simple explanation of the facts, and it was open to Dr. Haworth to explain them in any other way if he preferred.

The Authors replied briefly to the various questions asked and criticisms offered as follows: (Mr. Campbell) The observations show that the discrepancy, although very small, lay outside the limits of experimental error. It would be interesting to examine the behaviour of other instruments of the same and of different types. (Mr. F. E. Smith) Readings were taken continuously for half an hour or more, direct-current readings in opposite directions being made between those with alternating currents. The direct-current readings showed no systematic differences such as would arise from concentration changes. The latter would be produced at the main electrodes only, and could not affect the readings except indirectly by diffusion, of which the effects would be negligible in the apparatus used. (Mr. Whipple and Prof. Howe) The variation of the apparent contact P.D. with the voltage might have an electronic origin. (Mr. West) There was the same difference between the continuous and alternating-current effects even when the frequency was the lowest (about 10 ~ per second) that would give steady readings. (Mr. Addenbrooke) The apparent contact P.D. for a given voltage, in the electrometer used, changed very little from day to day. The variation with the voltage, to which they referred, was a different phenomenon. If the vibration of the needle produced any effect they would expect that effect to be different at different frequencies, but no evidence of this was found. (Mr. Worley) Kohlrausch and Holborn had given information sufficient to satisfy all ordinary needs in their "Leitvermögen der Elektrolyte."

Dr. Haworth was, of course, entitled to seek fresh explanations if those of other workers seem to him insufficient. In order to find whether the resistance of a conductor, metallic or otherwise, varies with frequency, it is necessary to avoid possible sources of error in the method of measurement. With

molecules in solution possessing the properties he suggests it would be remarkable if Ohm's law were obeyed by electrolytes under any circumstances. His next remark suggests that he has found since his first experiments with platinum that the apparent variation with frequency can be reduced by platinisation. To the authors this result presents no difficulty. If, however, Dr. Haworth's explanation is correct, new results should appear. The apparent resistance of an electrolyte should be alterable by merely placing pieces of platinised platinum in the containing vessel. Similarly, the freezing point or the boiling point of the electrolyte should be altered by the addition of this material. So also with mercury. With regard to the effect of heat, the opinion is very generally held that most of the results of other observers are best explained by making the opposite assumption to that which Dr. Haworth favours.

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